ECONOMICS
OF
ELECTRICAL DISTRIBUTION

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PREFACE

Electrical Engineering is more than the application of the principles of electricity to the design, construction and operation of a machine, a power plant or a distribution system to the end that it fulfills satisfactorily its intended purpose. It is highly desirable that efficiency be also accomplished. Further than this, it should be the aim of all engineers to develop efficiency in its broader sense, that is, by the realization of maximum economy in both design, construction and operation.

In designing, constructing, or operating an electrical distribution system, the object to be striven for is to provide all customers with a good quality of service at the least possible cost over the system as a whole. This result can be attained only through a careful and conscientious application of the principles of economics to all parts of the system. It is to define those principles and to indicate methods for their application that this book is presented.

This work has been prompted by the realization that the transmission and distribution system, representing a large part of the total investment in any central station and offering a wide field for economic study has not often been given sufficient attention. Much valuable information may be found scattered through the engineering literature, but it is thought that there is need of bringing together the factors involved in such a study and of discussing the subject as a whole.

This book is by no means an attempt to cover the whole field of economics as applied to the central station system. It is well recognized that the range of problems encountered is very broad and varied and that, as yet, comparatively little progress has been made in such work. It would be impossible in a work of this kind to cover in detail even the field of distribution and transmission economics. The purpose of the book is to present the need for the application of economic principles to the design of distribution systems, to explain the fundamental principles involved, to indicate the types of problems most often encountered and to offer methods of studying such problems and reaching their solution.

A considerable part of the material presented in this book has appeared previously in the *Electrical World.* The chapter on single-phase secondaries is a rearrangement of material published in the *Proceedings* of the American Institute of Electrical Engineers.

Merrill W. De Merit has furnished valuable assistance particularly in the chapters on Energy Cost and Underground Lines.

DETROIT, MICH.,
Dec. 1921.

THE AUTHORS.

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PART I

CHAPTER I

INTRODUCTORY

Electrical engineering design has three necessary components, *i.e.* electrical design, structural design and economical design. They are equally important and no electrical engineering problem can be solved properly without consideration of all three components in a thorough manner. The solution of any problem will come nearest to perfection as each of these forms of design is more soundly analysed and the combination of the three more intelligently applied.

Structural design calls for the study of materials used and their combination into the desired structures in such a way that proper factors of safety may be obtained. Electrical design calls for a knowledge of electrical phenomena and the application of this knowledge to achieving results satisfactory from an operating standpoint. Economical design calls for the knowledge of costs and their application to the determination of the most economical design possible. It is evident that there must be overlapping between these three fields. Structural design depends in most cases on the size of electrical apparatus, conductors, etc., which in turn is governed by the electrical design. Both structural and electrical features, on the other hand, should be planned with a view toward economy. Where there is a choice of more than one possible design which is satisfactory from a structural and an electrical standpoint, the decision should be based on study of the relative economy of all the alternatives considered. It is proposed in this book to consider electrical distribution from the standpoint of economy principally. There is no intention

however of minimizing the importance of structural and electrical design. Economical design has probably received less attention than the two other forms and the emphasis brought on it here may help to give it its proper place in the work of obtaining correct solutions to electrical distribution problems.

Scientific books in general may be divided into two classes. The first class gives facts and data obtained from practical experience. The second class gives methods and suggestions of means of attacking the study of certain problems. This book belongs to the second class. In work with distribution systems the conditions met with in different localities are diverse. Costs of material and labor and methods of construction and operation are varied and changeable. It follows that if definite figures and results are given which might apply to one system at one particular time, these same figures might be very misleading if applied indiscriminately to any other locality, system or period of time. It is necessary, therefore, that any figures given here, particularly those referring to costs, should be considered only as examples of the methods presented and not as material on which to base any calculations.

Furthermore, there is no intention to cover fully either in detail, or in a general descriptive manner all the problems, which might be encountered in electrical distribution, viewed from the economic standpoint. It is desired to present some general methods that can be applied to the solution of most of these problems, to give examples of the application of these principles to some ordinary cases, and to indicate the types of questions most often arising in this work.

The book is made up of two parts. The first part including Chapters II to VII is intended to give working methods, or so to speak, to furnish the tools to be used in solving electrical distribution problems economically. The authors have tried to present the underlying principles which are the basis of economic study of this kind. The fundamentals are not new but their application to the design of a distribution system is not so generally understood. The use of accurate annual costs instead of first costs (or loose estimates) as a basis for determining the most economical installation goes back to Lord Kelvin and further. But, we see many engineers today ignoring the economical factors of the problem entirely and others basing all comparisons on first cost only, or attempting to apply the

so-called Kelvin's Law indiscriminately. A knowledge of the fundamentals is absolutely essential if any such study is to be worth the time spent upon it. Otherwise the engineer may be guilty of deluding himself and others with figures which are entirely inapplicable to the case in hand.

Chapter II attempts to create a point of view regarding the subject at hand. Then in succession are taken up means of analyzing and tabulating costs of material and labor, of determining the cost of energy to be used under various conditions, of studying the characteristics of loads to be handled, and of forming general equations for solving problems in economical design. Finally a chapter on power loss and voltage drop is included to provide convenient means of handling these most important electrical phenomena which must always be considered in connection with economical design.

The second part of the book consists in presenting the application of the methods, or the use of the tools described in the first part. For convenience a division of the subject has been made into transmission lines, power circuits, lighting circuits, secondaries and underground lines. Under each heading some of the general problems encountered are indicated and some particular problems are solved in detail as definite examples. Special or unusual problems have not been considered, the work for the most part holding to the everyday questions met with in practice. Chapter XVI in conclusion, touches on a few of the general problems which apply to the system as a whole, such as the location of generating station and substations, the relation of one part of the system to the remainder, etc. Chapter XVII takes up briefly a few of the problems in distribution pertaining to industrial plants.

It will be seen that the possibilities for study in the field of distribution economics are boundless. The deeper one goes into the subject the more problems present themselves and the more evident becomes the need for careful and accurate investigation. The objection may be raised that the changeable character of the loads carried, and the fluctuations of prices make a too detailed study of economies impracticable. That is no doubt true sometimes, as regards individual problems. In the long run, however, the knowledge gained is always valuable and of great assistance in deciding general policies. As our information on the behavior of materials and on the characteristics of the various types of loads becomes more accurate, the practicability

of the application of detailed economies will increase. With the increasing demand for efficiency and economy in all lines of endeavor, the necessity for the economical operation of distribution systems becomes more imperative. In any case, an exact knowledge of the true cost will always lead to efforts to reduce that cost if possible.

CHAPTER II

APPLICATION OF ENGINEERING ECONOMICS TO TRANSMISSION AND DISTRIBUTION PROBLEMS

As stated in the first chapter, the purpose of this book is to present some of the problems encountered in the design of lines for transmitting electrical energy and to approach their solution from an economic point of view. More specifically it is purposed to apply engineering economics to distribution- and transmission-line layouts indicating how the most economical installation consistent with good service can be determined.

No argument should be required as to the advisability, or rather necessity, for the application of economic principles to engineering design. Engineering should make for efficiency but there can be no real efficiency unless there also is accomplished economy. Economy cannot often be recognized at first glance. A working knowledge of its fundamentals at least is required. Engineering and engineering economics are therefore synonymous in all problems directly involved in the production or distribution of a commercial commodity. This includes the great majority of our present everyday engineering problems and naturally those of transmission and distribution.

It is not within the scope of this book to consider unusual problems such as very high-voltage transmissions for example, or installations of special apparatus whose use has not become a matter of accepted practice. Rather, the general purpose will be to make a few analyses that will indicate methods of finding the most economical design under usual conditions with material of known characteristics and at known prices. It is especially for the improvement of the design of the large class of everyday jobs that this work is written. Such analyses will, naturally, not only indicate the most economical installation with materials and construction methods already in use but will make it possible to determine where any economy can be effected by changes in these methods or materials. It must be recognized that the

majority of the transmission and distribution lines have been laid out by rule-of-thumb method, with no definite conception of actual costs or economy. The methods here presented have been developed by the study of actual problems encountered in practice. It is hoped that these methods will on the one hand show the magnitude of the savings possible by the use of engineering economics and on the other hand be simple enough in their application to be considered worth using in many places where more empirical methods are used today.

METHOD OF TREATMENT

Practically all such problems of engineering design require treatment in three distinct stages for a complete solution. At first all the data must be obtained that can be determined with exactness. Such elements as material costs, strength of parts of construction, load tests, etc., are here included. Second, those elements which are not subject to exact measurement or computation must be decided upon. These are usually a development from practical experience over a number of years and are often matters of accepted standard practice. They are such items as probable increase in load, allowable voltage regulation, safety factors, standards of construction, etc. Empirical methods must here be employed. The third element is the most intangible—the good judgment of the engineer based on his knowledge and experience and applied to the particular problem in hand to so utilize the first two elements as to create an efficient and economical design best adapted to the situation. All these elements in their proper proportions are equally important in attaining the most satisfactory solution for any problem. Empirical methods if applied blindly may lead far away from the purpose for which they were originally intended. Judgment based on experience alone is liable to become a mere guess unless supported by exact knowledge and good practice. It would appear therefore, that the further the exact data can be carried in a problem the less dependence need be placed on the more intangible elements and hence the greater certainty of the best solution. The extension of this exact knowledge is the chief purpose of the study of the economic features of a problem. In many cases economy may be made the deciding factor between two or more designs apparently equally good from other points of view.

Exact Data.—For any given system there are a number of limiting conditions which reduce the unknown factors in the design. There are certain standards such as transformer sizes, wire and cable sizes, etc., which are established by the manufacturer. Other limitations are established by the accepted good practice of the profession in general. Also each company has certain standards of materials and construction to which it is wise to adhere unless there is a clear advantage in making a change. The most complete information obtainable on all such data relating to the particular system in hand should be kept available for ready reference. Also other items which can be obtained exactly for only the one locality or company such as costs of material, labor and energy should be determined as accurately as possible and be revised from time to time to conform to changing prices, wage scales and costs of production. This information can be readily kept up to date and tabulated or drawn up in curves. Then only such factors as are inherent to the particular line itself need be determined in making the design.

Empirical Data.—Some of the empirical elements of the problem of transmission and distribution of energy are worthy of considerable attention. One of these is the element of "good service." It is understood that in any design or layout the greatest economy "consistent with good service" is the object. Just what is good service may be a matter of considerable question.

Good Service.—Good service is not dependent on the ideas of the engineer or the "server" as to what it should be. Service is good when it satisfies the one that is served. That man who at the receipt of his bill feels that he has gotten all that was due him is receiving good service. He is willing to exchange his money for what he has received from the server.

The human element is therefore the most important one in determining good service. Customers will expect as good or better operating conditions than they have been in the habit of getting. Companies have, so to speak, educated their public to certain expectations and they must live up to such expectations. It is evident however that it is impossible to eliminate all interruptions or conditions of poor regulation. In thickly populated districts with a high-load density, practically continuous service at good voltage is easily given. In outlying, scattered

districts the cost of the same quality of service would be prohibitive. From the company's point of view the service must be of good enough quality to insure earnings and yet not so good as to necessitate too large expenditures. It is therefore necessary, before attempting to solve a transmission or distribution problem economically, to ascertain what quality of service is required both from the customer's and the company's point of view.

Good service can therefore only be determined in a general way and will vary with locality, company, kind of load, etc. It might be said, however, that with the modern refinement of methods and equipment and the nearly universal high requirement of the customer we are approaching more and more a condition when wide variation of voltage and long or frequent interruptions will hardly be permissible.

Increase in Load.—A second quantity which must be empirically determined is that of load increase. The variation of the loads to be carried and the growth of energy demand that is prevalent in nearly all localities are always matters for considerable study in connection with a layout. There are some cases where it is possible to design for a certain demand that can be assumed to remain constant for a period of years, possibly for the life of some part of the equipment. Such would be the case in planning lines to a large power installation where growth could be taken care of by new circuits. On the other hand such load as house lighting increases continuously and its increase can only be estimated from the figures for the past few years and general business conditions. In any case we must design for a system that will be most economical over its useful life, in other words when the sum of the annual costs for all years under consideration will be a minimum. It is further necessary to study as exactly as possible the number of years to be cared for by the present design and minor changes that can be made at various times during that period for taking care of the changes in load. Generally the design should cover the expected life of that part of the equipment whose life is shortest and which would require a large expenditure for its replacement. Any considerable change necessary to care for larger load could then be made at the same time with relatively smaller cost. In some cases, however, where the probable increase in load is more definitely known, the economical life of the present design can be accurately determined even well within the expected life of all the important parts.

Financial Conditions.—The condition of the money market may be another determining factor in an economical design. Often the difficulty in securing necessary funds for a job may require the engineer to spend less at the time of installation than is consistent with economy over the period the design covers. However the condition should be carefully considered in order to still obtain the best design as limited by the money available.

Relation of Parts to Whole System.—The fact that the transmission or distribution line is only one part of the system transmitting energy from the turbine to the customer must be kept in mind. Its design will affect and be affected by existing or planned conditions in the other parts. It is therefore always necessary to treat a line not only as an independent unit for some specific purpose but also as a working part of the whole system.

Bearing in mind the general considerations brought out above, the table given below will show the most important points to consider in studying the layout of an economical line. The following chapters will discuss in more detail the methods of obtaining this data and its application to particular classes of problems such as transmission lines, power lines, secondaries, etc.

1. Load	Location.	
	Present size.	
	Probable increase and rate of increase.	
	Characteristics:	Phase requirements.
		Power factor.
		Variation, daily, weekly, seasonal, etc., and relation to station variations and peak.
		Unbalance factor.
		Maximum.
		Load factor.
2. Route of line	Investigation of advisability of possible routes, affected by:	Available pole or duct space.
		Purchase of right-of-way.
		Difficulties in construction.
		Interference with or from other lines, physically or electrically.
		Convenience of operation.
		Possible future extensions.
		Effect on system as a whole.
		Cost as compared with other possible routes.

3. Physical constants	Length of line Spacing Voltage: desired, available. Resistance of line Reactance Voltage drop Regulation: desired, available	Under various line conditions as limited 1. By good operation. 2. By standard practice. 3. By routing. 4. By economical considerations. 5. By good service.
4. Costs	1. Determination of labor costs, material costs, and energy loss costs	For all conditions considered
	2. Application of those costs to determine the best line for the purpose.	
5. Relation to System	General considerations that may affect most economical line found above under (4) due to its being a part of a large system. Financial considerations.	

From inspection of this table it is seen that the five divisions in it are in one way or another interdependent so that each one must be considered in relation to all the others. It is only by a careful study of the problem from all angles and a coordination of results that a satisfactory solution can be obtained. In this book, however, the economic features of the design have been emphasized rather than the mechanical or electrical. These latter have been covered quite thoroughly in other works.

CHAPTER III

COSTS

Principles Underlying the Determination of True Costs—Formulas for Unit Labor Costs—Cost Records—Annual Costs

The fundamental basis underlying the whole question of economic design is the accurate determination of costs, *i.e.*, costs of construction and cost of energy. If these are not correctly determined any conclusions drawn from their use will have little value. Further it is generally inadvisable to accept for this purpose any cost data which has not been locally derived, as every power company has its own methods and standards of construction, its own labor costs, its own efficiencies of operation. These may differ widely. Hence, it is absolutely necessary to make as complete a determination of local costs as possible as a basis for any economic study.

Evidently when it is decided to establish a cost record on construction, material and labor, difficulties will be encountered in obtaining correct information regarding all details. Much valuable information will be found in appraisals and much can be gotten from the accounting records by a skillful economist. However, in most cases it will take several years to establish a complete record and special studies will be necessary. This record, when obtained, is an easy thing to keep up to date and in convenient form for use. In the meantime certain makeshifts will be necessary. If a complete record of unit construction and operating costs is not obtainable or when lack of time or of available records requires any item of cost to be estimated, the estimate should be based on known conditions as far as possible. Further than this there should be determined by what percentage the final result will be affected by any reasonable variation in the assumed cost. For example, the cost of energy at any point may be assumed to be 1 ct. per kilowatt-hour. If this is not an accurately determined figure however, it would be well to determine how the result would be affected if the cost of energy were say ¾ ct. or 1¼ cts. per kilowatt-hour. Such a comparison will at least place the solution of the problem within definite limits.

It is a generally recognized fact that the computation of first cost is only a step in the investigation of the real cost of a proposition. The economy in comparison with any other alternative proposition can only be determined when the annual cost is investigated. The annual cost must include all annual charges against the investment itself and all operating costs, maintenance, repair and energy losses. Of course annual costs cannot always be considered the determining factor, since on items involving large expenditure the possible difficulty of obtaining capital may have considerable weight. Usually, however, annual cost may be accepted as the criterion.

Before annual costs on a piece of property such as a transmission line can be obtained it is necessary to determine its total value or first cost.

First Cost.—The determination of cost figures for general use is greatly facilitated if line construction, materials and methods are standardized. This allows a fairly accurate figure to be obtained for standard units as per pole, or per 1,000 ft. or per mile for any type of construction. Otherwise smaller units must be depended upon such as per crossarm, per insulator, per 100 ft. of wire, etc. In any case it will be found exceedingly valuable to have as complete a record as possible of itemized costs from the smallest part, such as a bolt, up to an average cost for a large assembled unit such as per mile of line. This should include both material and labor costs with overhead expense all shown separately. Provision should also be made for easy revision of costs as prices change. This revision should be frequently made when prices of labor and material are fluctuating to any extent since a large part of economic study deals with possible new construction which will be at present or future prices or with old construction which represents a value equal to new construction less a certain percentage for its age. Certain types of problems require the consideration of the actual cost of the old construction at the time it was installed but these are not so general.

There are a number of items which must be included in addition to current quotations on material or actual labor time in erecting. To material price may be added such items as freight, treating material for poles, tie wires on line, etc. To actual unit labor costs should be added a proportional amount for unoccupied time, rainy days, vacations, transportation inspection, etc. For example:

No. 2 Solid wire	Weight per mile, one wire, pounds	Cost at 21 cts. per pound	Freight on returned reels	Injury to reels	Ties per mile	Total per mile	Per cent of wire cost
	1,066 lb.	$223.86	1.86	2.32	3.63	$231.67	103.4

Naturally these additional items vary for each unit and each locality. It is usually possible to obtain a percentage which may be added to actual price to include such incidental material.

Loading.—The item of overhead expense or loading is one which must be included in nearly all problems involving costs. There are various methods of applying this loading but for the purpose under consideration the method of unit loading is probably the most satisfactory, *i.e.*, apportioning to each individual item its pro rata cost for different items of overhead expense, both on material and labor. It is evident, for example, that the item of breakage will be greater for insulators than for wire. The items which may legitimately be included in loading are as follows:

Waste—end trimmings of wire, cable cut back for splicing, incompetent labor, etc.

Loss and breakage—theft, broken insulators, etc.

Tool expense—tools used up, broken, stolen and repairs and depreciation on them.

Direct supervision—engineering, heads of departments, general foremen, office expense, clerks, stenographers.

Injuries and damages—doctor and hospital expense, liability, insurance, etc.

Purchasing expense—all expenses of purchasing department.

Stores and supplies expense—all expenses of stores department and handling.

The percentage to be applied to any unit to cover any one of the above items will depend entirely on local conditions and no figures could be given here which would be of any value. The total loading percentage is usually between 10 and 25 per cent depending upon the class of property and upon local conditions.

As will be seen later all problems do not require a detailed application of loading. Many problems involving the comparison of relatively small amounts of construction may be safely considered from the point of actual material and labor costs

alone, considering overhead to be equal in both cases. The question of whether or not to apply loading must be determined by the conditions involved in the particular problem under consideration. In general where good cost records are kept it will be just as easy and more accurate to include it in all problems.

Labor costs on any unit may usually be reduced to an equivalent formula of man hours for various classes of labor as foreman, lineman, groundman, etc., for each unit plus a percentage for incidental expenses as above. Such formulas facilitate revision of prices when necessary as the current wages may be substituted and the labor cost on the unit easily obtained.

For example, if a gang consisting of 1 foreman, 1 truck, 1 chauffeur, 4 linemen and 3 groundmen can string 1.6 miles of No. 0 bare stranded wire per day, on an average, the cost per mile of wire will be .728 times the cost of the gang per day

$$.728(1F + 1T + 1Ch + 4L + 3Gr)$$

where the symbols represent the daily wage of the various classes of labor included, foreman, truck, chauffeur, linemen and groundmen.

Some of the labor formulas are more complicated but all are computed on the same basis. For poles, for example, several different gangs are included in the labor charge, *i.e.*, unloading, distribution, framing and roofing, digging hole, and setting and the gangs for each operation may be different for different sizes of poles. A formula similar to the above may be determined for each operation and the total labor formula for the pole is a composite of these. For example:

if G_1 represents the daily cost of the unloading gang	$=(1Tr + 1Ch + 3Gr)$,
G_2 represents the daily cost of the distributing gang	$=(1Tr + 1Ch + 3Gr)$,
G_3 repres nts the daily cost of framing and roofing	$=(\frac{1}{4}L + 2Gr)$,
G_4 represents the daily cost of the digging gang	$=(\frac{1}{7}F + \frac{1}{7}Tr + \frac{1}{7}Ch + 1Gr)$,
G_5 represents the daily cost of the setting gang	$=(\frac{5}{7}F+\frac{5}{7}Tr+\frac{5}{7}Ch+5Gr)$

(The $\frac{1}{7}$ F, $\frac{5}{7}$ Tr, etc., are occasioned by the fact that one foreman and one truck serve several gangs at one time.)

Then the total labor cost on one 35-ft. pole equals

$$.0160G_1 + .1403G_2 + .063G_3 + .2243G_4 + .0468G_5$$

Cost Records.—For a complete record of line costs there are necessary the following items:

(*a*) Current prices of material and labor of all kinds.

(*b*) Labor formulas and constant multipliers for various classes of materials with loading percentages for both.

(*c*) Current material and labor costs on units of construction, as per cross-arm, per pole, per insulator, etc.

(*d*) Current costs on assemblies. Assemblies may range from small items such as a crossarm erected with braces, bolts, etc., or a ground connection with wire, ground rod, wood moulding, etc., up to large items such as cost per mile of transmission line on cost of a railroad crossing, etc.

Some examples from such a record are given in the following:

(*a*) *Prices* (*at warehouse*)[1]

No. 3 Porcelain insulators	$.16 each
No. 2 T. B. W. P. wire	.2348 per pound
Primary fuse boxes	3.94 each
30-ft. 6-in. pole, rough	7.28 each

etc

(*b*) *Labor formulas.*

Wire stringing (*single wire*) Gang $= (1F + 1Tr + 1Ch + 4L + 3Gr) = G$

	Miles per day	Labor cost per mile without loading	Plus 16.55 per cent for loading
No. 6 Solid	2.8	.3575G	.4165G
No. 0 Stranded	1.6	.625G	.728G

Ground connections Gang $= (1Tr + 1Ch + 3L + 1Gr) = G$

	Labor cost each without loading	Plus 18.75 per cent
23,000-volt ground connections	.02075G	.0248G

Anchors and guys Gang $= (2/7F + 2/7Tr + 2/7Ch + 2L + 1Gr) = G$

	Labor cost each without loading	Plus 17.65 per cent for loading
Pole to pole one 3/8 in	.0833G	.098G
Pole to anchor two 3/8 in	.2833G	.3333G
Stub to anchor one 1/2 in	.25G	.2941G

[1] The figures given below must not be taken as representing current prices. They are given for example only.

Crossarms Gang = 1/3*F* + 1/3*Tr* + 1/3*Ch* + *L* + *Gr*) = *G*

	Labor cost each without loading	Plus 17.65 per cent for loading
23,000-volt, single-arm, 36 in.	.04*G*	.047*G*
23,000-volt, double-arm, 64 in.	.131*G*	.154*G*
23,000-volt, double-arm, 64 in. (strain insulators, both ways) etc.	.138*G*	.162*G*

(*c*) *Unit costs.*

	Material	Plus per cent. loading	Labor	Plus per cent loading	Total
		(21.15)		(16.55)	
3 No. 6—Secondary, 1,000 ft.	$99.00	$120.00	$34.35	$40.00	$160.00
2 No. 4—Primary, 1,000 ft.	91.40	110.80	25.46	29.70	140.50
		(22.64)		(17.65)	
3¼ × 4¼ × 92 in., six-pin crossarm and hardware	.74 .54	1.64	1.04	1.22	2.86
		(21.80)			
No. 3 porcelain insulator	.16	.21	Labor included in cost of stringing wire		
		(26.08)		(20.95)	
Pin—1¾ × 10⅛ × 1 in.	.06	.076	.10	.48	.196
		(16.91)		(12.16)	
Poles—40 ft. 7 in. etc.	16.70	19.50	10.26	12.48	31.98

(*d*) *Assemblies.*

15 *kva.* Sϕ transformer installation

Pole material		Plus loading, per cent	
9—No. 3 Porcelain insulators at .16	$ 1.44	31.80	$ 1.90
6—Glass insulators at .043	.258	31.80	.34
15—Screw brackets at .22	3.30	26.08	4.16
2—Six-pin crossarms at .74	1.48	22.64	1.82
2—Blocks, at .48	.96	22.64	1.18
6 Braces at .1275	.765	22.64	.94
10—3/8-in. bolts at .02	.20	22.64	.245
4 3/4-in. bolts at .12	.48	22.64	.59
3—Lags at .038	.114	22.64	.14
2—Primary fuse boxes at 3.94	7.88	22.64	9.67
2—Secondary fuse boxes at 1.00	2.00	22.64	2.45
2—Lightning arresters 4.86	9.72	22.64	11.91
Ground-rod, cap and moulding	1.58	23.79	1.95
5—lb. No. 6 wire	1.395	21.15	1.69
6.6—lb. No. 4 wire	1.78	21.15	2.16
30—ft. 5/16-in. galvanized-iron wire	.525	22.64	.63
Staples, screws, etc	.17	22.64	.21
Total	$34.05	.....	$ 41.99
Labor 1/3(1*F* + 1*Tr* + 3*L* + 1*Gr*) = $13.90 plus 15.45 per cent loading			16.10
			$ 58.09
15-kva. transformer $185.90 plus 16.91 per cent			217.00
Total			$275.09

Cost per 1,000 ft. of line with 125-ft. span.

	Cross-arms, one per pole	Pins	Insulators	Wire	Wire, pins, insulator	Plus cross-arms	Plus 35-ft. poles
Primary							
2 No. 6	$22.80	$3.14	$3.36	$106.80	$113.30	$136.10	$335.22
2 No. 2	22.80	3.14	3.36	186.15	192.65	215.45	414.57
Secondary							
3 No. 4	22.80	4.70	1.37	210.00	216.87	239.67	438.79

The above are merely examples selected here and there and in no way indicate the complete record. It is easily seen that the compiling of such a record is a matter of considerable labor and investigation. Once obtained, however, in this form a revision is a comparatively simple matter.

While the costs thus given are average figures, especially for labor, they may be applied in the locality in which they were derived without great error for estimating individual jobs unless some unusual field conditions indicates extraordinary additions to some part of the cost. For economic studies of course average figures are usually desirable.

ANNUAL COSTS

The determination of annual costs as used in this work is naturally divided into two parts, that pertaining to the physical property itself such as construction and maintenance costs and that pertaining to the load carried, *i.e.*, cost of energy.[1]

Annual Costs on Physical Property.—The items to be considered under the first heading, *i.e.*, annual costs on the physical property, will each be discussed briefly. They include, interest, taxes, insurance, maintenance, repair and depreciation.

Interest.—Whenever money is invested in a piece of property a legitimate rate of interest must be expected as part of the earnings of that property unless it be run at a loss. Interest must be figured on the total investment involved including all material, labor and overhead costs. The rate at which interest should be charged may vary with the problem under consideration. Fundamentally it should be the current rate of interest on sound investments or the rate at which the company could borrow money under ordinary conditions. Sometimes the average rate paid on total capitalization may be taken but in case the dividend on capital stock is fairly large, part of it might be considered as a profit in excess of a fair rate of interest. In some cases due to poor financial conditions or for an emergency a company might have to pay a higher rate for money, even on bonds, than the market rate. All such factors should be considered in determining the interest rate to be used.

Taxes.—Taxes are an ever-present charge on any property and usually the definite percentage may be easily determined from the company's accounting records.

[1] For determination of "Energy Cost" see Chapter IV.

Insurance.—Insurance against loss by fire is the most common form but on some classes of property insurance against theft, storm, etc. is also carried. In any problem the kind and amount of insurance chargeable to each class of property should be investigated. In many cases no "Insurance" charge is necessary.

Maintenance and Repair.—Maintenance and repair will be different for each unit considered. No two transformers for example will require the same amount of attention during their life even though similarly located; breaks in a line can rarely be anticipated, etc. A large part of maintenance is occasioned by imperfection in material. Maintenance and repairs due to such causes on any individual piece of equipment cannot be foreseen. In such cases average figures only can be obtained from actual experience over a number of years. Other items of maintenance such as inspection, testing, etc., can be quite definitely determined from payroll and time reports.

Depreciation.—No detailed discussion of depreciation can be here included. Strictly speaking, depreciation is the percentage by which a piece of property is reduced in value each year of its life (by value is not meant necessarily selling price). From an accounting point of view, on the other hand, a certain amount must be set aside each year to replace the property when worn out. Sometimes the usefulness of the property in service is considered as a measure of its value. All these different viewpoints give rise to different methods of figuring depreciation, and a discussion of these may be found in other works. For the purpose of this work however what is known as the straight-line method is probably the simplest and most satisfactory. This method considers a piece of property to have given service for the years of its life for a definite total cost which may be equally divided between the years. The cost is obtained from the total first cost of the property in place including material, labor and overhead charges, less the salvage value at the end of its life, plus the labor cost necessary to salvage it. This cost is divided by the estimated number of years of life and the percentage of depreciation taken as the percentage of the total first cost thus obtained. Naturally this will vary with different classes of property. Some will have little if any salvage value such as crossarms, for example. A pole, when rotted at the base, on the other hand can be sawed off and used again either as a shorter pole or a stub. Bare copper wire will have practically no physical depreciation,

the cost of stringing and removing together with tie wires and other incidentals making up the depreciation.

The years of life to use for any unit may depend on other things than its own life. For a transmission line, for example, it is probable that no definite limit can be fixed at which the whole line must be replaced. Poles and crossarms will be replaced from time to time when necessary as long as the line is in service. In such a case it is probably simplest to assume a definite life for the whole line, possibly the assumed life of a pole, and figure depreciation on all units, wire, insulators, etc. on that basis. Thus any class of property may have a different percentage of depreciation depending on where it is used.

A simple example of a computation for depreciation on insulated wire would be as follows:

Assuming new wire at 30 cts., scrap copper at 20 cts. per pound.

Cost of No. 0 wire per 1,000 ft.-420 lb. at 30 cents =	$126.00	
Labor of stringing (assumed)........................	10.00	
Labor of salvaging (including burning off insulation)	10.00	
Total..	$146.00	
Salvage value 319 lb. at 20 cts. per pound...........	63.80	
Net Cost....................................		$82.20

If the assumed life of insulation is 15 years

$$\frac{82.20}{15} = \$5.47 \text{ per year } \frac{5.47}{136.00} = 4 \text{ per cent per year.}$$

The matter of obsolescence which is sometimes considered as a separate figure may, for this work, be considered as a part of depreciation. Where it is anticipated that materials will become obsolete and require replacement before worn out on account of improvement in design, the assumed life and salvage value used in computing depreciation should be adjusted accordingly.

As an example of the above, the total percentage of annual charge on a pole line may be taken as 15 per cent, made up as follows:

	Per Cent
Interest..................	7
Taxes.....................	2
Insurance.................	0
Depreciation..............	6

Many special problems arise in figuring annual cost. One which also includes the idea of obsolescence is encountered when

the replacement of a serviceable line with one of larger capacity is being considered. The value remaining in the old line must be included in the computations and the total investment represented in the new line must include, in addition to the cost to build it, the present value of the labor necessary to build the old line, *i.e.*, first cost for labor less depreciation for years of age and the labor cost necessary to dismantle the old line. Such problems will be discussed in more detail later.

CHAPTER IV

ENERGY COST

Principles and Methods Involved in the Determination of the Cost of Energy and of Energy Losses

Since any economic study of transmission or distribution is, fundamentally, a consideration of the cost of energy or energy losses as compared with other costs (in general, fixed charges increase as energy loss decreases), it is of the utmost importance that the cost of energy be accurately determined. The value assigned to the unit cost of energy may be the deciding factor in a problem. This value, moreover, may vary considerably according to the assumptions, methods and precision employed in its calculation. It is evident that the cost per kilowatt-hour of the energy used by a 5-h.p. motor running 1 hr. per day, 25 miles north of the generating station may be quite different from the unit cost for residence-lighting load, 5 miles west of the station and both of these may be far from the average unit cost over the whole system. The question of the determination of energy cost offers an extensive field for study and one that generally has been only touched upon. This chapter will give some of the fundamental principles involved, a few methods of attacking the general problem and suggestions as to conditions governing variations in the general cost as applied to particular uses.

In studying energy cost, the fundamental difference between the cost of energy for rate making purposes and the cost of energy for economical study must be recognized. In determining the cost of energy for the purpose of adopting a rate scale, it may be sufficient to consider the system as a whole and determine the average cost per unit for each of a few general classes of load which have markedly different characteristics. The chief point to be kept in mind is the amount which the customer pays, and that this, on an average should at least equal the expense of the company, plus a reasonable profit. Usually about the same rate must be applied to similar customers within a reasonable area, unless there is a marked difference in the individual cost of serving each. There are also other factors than actual production and distribution cost which must be considered in rate making, such as public opinion, regulations and franchise agreements, previous practice, competition, etc. On the other hand, in making an

economic study, we are interested in the actual amount which the energy delivered to the point under consideration is costing the company and hence, how much money, if any, can be saved by reducing energy losses. If energy costs more per unit 5 miles from the station than it does 1 mile, for example, the amount saved will be correspondingly more important. For this purpose, then, it would seem desirable to investigate the cost of energy as fully as is warranted by the amount and accuracy of the information available on costs of construction and operation and on loads carried.

Even with fairly complete data at hand it is a difficult matter to determine definite figures for energy costs. The cost is affected by many quantities of a variable nature and these limit the extent to which it is practicable to carry the study. At any given point the cost of energy is largely dependent on the fixed charges and operating expenses of the central station and of the distribution system between the station and that point. It is also affected by the size and characteristics of the load at the point in question in relation to all the other loads on the system. Hence, strictly speaking, energy cost may be conceived as having a different value at every point on the system and for every different load at any given point. Loads of the same type may show a different unit cost according to their size and the unit cost may vary at different times during the day or even for different parts of the same load. The length to which the determination of cost of energy might be carried is almost infinite. In practice it will depend not only on the accuracy of the data available but also on how this cost is to be used.

Classification of Costs of a Central-station System.—The various items entering into the total annual cost of a central station may be classified as follows:

1. Costs dependent on the number of customers.
2. Costs dependent on the peak load carried or the demand.
3. Costs dependent on the total output in kilowatt-hours during the year.

This method of classification which was suggested by Dr. John Hopkinson in England in 1892 and was later enlarged upon by H. L. Doherty and other writers, is quite generally accepted as sufficient for ordinary purposes. It must not be assumed that all expenses come strictly under one of these three classifications. There are a great number of other minor divisions which

might be made. For example, there are certain operating expenses at the station which are dependent on the efficiency and size of the machines and the relation of the load at any time to the capacity of the generators in use and to the method of operating the station and the system. It does not appear practical however to attempt to include all such classifications. The above three are the ones of most importance and other costs can be included in one of them without great error.

Consumer's Cost.—The first division, consumer's cost, usually need not be considered in determining energy cost for economic study, as it is independent of the amount of load carried. This cost occurs beyond the limits of the lines and need be added only when determining the charge to the customer. Care must be taken however that charges actually belonging to this classification are not included under either of the other heads. Here rightfully belong the greater part of charges for general-office expense, sales expense, metering, billing, collection, etc., part of cost of service wires and a percentage of various other charges according to local conditions.

Demand Cost and Output Cost.—The other two cost divisions, demand cost and output cost, must include all charges which are dependent on the load carried. The demand cost is that part of the total cost which is caused by the fact that the system is built and operated to care for a certain maximum load. If this peak load is 100,000 kw. the station must have a capacity of at least 100,000 kw., with a reasonable amount of reserve, regardless of the fact that that capacity may be reached for only a short time each day. The output cost or kilowatt-hour cost on the other hand is that part of the total cost which is occasioned by the total output in kilowatt-hours regardless of the *rate* of that output.

Some items of the total annual cost clearly belong only to demand cost while others which might seem to depend only on output have some percentage of demand cost included. In the first group come interest, taxes, insurance, depreciation, etc. on the generating-station building, interest, taxes, etc. on boilers, turbines, generators and other equipment and a large part of their depreciation and maintenance. Also fixed charges on lines and substation equipment are here included. Under the second group comes part of the cost of coal, oil and other expendable materials, part of the labor of operating, also part of the energy

losses on lines and transformers due to the fact that they are kept energized at all times. The apportioning of such costs is a matter for considerable study. The method of operation may effect the amount chargeable to demand in certain cases. Large machines are less efficient at small loads. Hence if large units are employed and are run far below their most economical load for the greater part of the day, the extra expense due to decreased efficiency may be charged to demand. In a large, efficiently operated station this condition would not occur to any great extent, but it suggests some of the items which must be considered. In fact there must be included in demand cost all charges directly or indirectly occasioned by the total capacity of the system. The remaining annual costs, aside from the consumer cost before mentioned, may be considered as dependent on the output. These comprise the kilowatt-hour cost.

Methods of Making Classification.—The actual division of the total annual cost on any part of the system into three classifications will depend largely on local conditions. The relative percentages will probably be different for each company. Several general methods of attacking the problem are in use. In the generating plant, for example, an empirical analysis can be made of each item of cost, such as charges on building, on steam equipment, on electrical equipment, fuel, labor, etc. The proportion belonging to each classification may be estimated from known conditions, keeping in mind the general definitions of demand cost, kilowatt-hour cost and consumer cost. One method of separating demand and kilowatt-hour charges on fuel, lubricants and such items is by comparing costs under a period of light load and one of heavy load—a month at different seasons of the year. For example,

if C_D = demand cost, per unit demand
C_f = kilowatt-hour cost,
D = station demand,
F_1 = kilowatt-hours at low period,
F_2 = kilowatt-hours at high period,
C_1 = total cost at low period,
C_2 = total cost at high period,

$$C_1 = C_D D + C_f F_1$$

$$C_2 = C_D D + C_f F_2$$

$$C_f = \frac{C_2 - C_1}{F_2 - F_1} \qquad C_D = \frac{C_1 - C_f F_1}{D}$$

This should give satisfactory results if there were enough difference between the loads at the two periods to give a good comparison of cost.

Another somewhat similar method,[1] that may be applied to items such as total station or system costs, which involve all three classifications, considers the total cost over three given periods, three years for example or three different months. Using similar symbols to the above with

$$Cc = \text{customers cost},$$
$$N = \text{number of customers},$$
$$C_1 = C_D D_1 + C_f F_1 + C_c N_1$$
$$C_2 = C_D D_2 + C_f F_2 + C_c N_2$$
$$C_3 = C_D D_3 + C_f F_3 + C_c N_3$$

If these are solved simultaneously the values of C_D, C_f and C_c can be determined.

An example of a typical division of costs between demand and kilowatt-hour of some of the items for the generating station is as follows:

	Demand, per cent	Kilowatt-hour, per cent
Superintendence	100	
Wages	90	10
Fuel	25	75
Lubricants	25	75
Station supplies, etc	100	

These general methods may be adapted to other parts of the system, transmission lines, substations, etc., as well as to the generating plant. In analyzing the cost of a hydro-electric plant, the available supply of water is an important factor in the consideration. It is evident that the kilowatt-hour cost will depend considerably on whether the supply is unlimited or whether storage is resorted to for regulating the flow.

It appears then that the first step in the study of energy cost is the determination as accurately as possible of the total annual costs on each subdivision of the system and the proportion of these costs, in each case, belonging to demand and to output. Naturally the more detailed the accounting record on various parts of the system, the easier the determination of these costs

[1] This method is more fully explained in "Central Station Rates in Theory and Practice," by H. E. Eisenmenger, *Electrical Review*, Vol. 75, Aug. 23, 1919, p. 305.

will be. If an entirely new system is being considered, the various quantities can be only estimated from data of other similar systems and present prices on construction materials, equipment, etc. For the purpose under consideration, if the study of energy cost is to be carried to any degree of exactness, it is probably better to prepare separate figures on each of the three classifications of costs as related to different parts of the system, generating station, various substations, underground-cable lines, overhead power lines, feeders, etc. Just how much of such detail is necessary will be determined by the method which is to be used in apportioning the costs among the various classes of loads and localities.

Apportioning of Demand and Output Costs to Various Types of Loads.—When the proportions of the total cost chargeable to demand and to output have been determined, there arises the problem of finding what part of that demand cost or of that kilowatt-hour cost belongs to any particular load or type of load. The *Kilowatt-hour cost* may be simply disposed of for the present by considering that the kilowatt-hour charge at any point in the system is equal to the sum of the kilowatt-hour costs incurred on all parts of the system from that point back to the generating station. Considerable study may be involved in determining the kilowatt-hour costs on such parts as transmission lines or power lines, since a great part of the cost depends on the energy losses and the cost of these losses in turn includes the demand and kilowatt-hour charges up to that point. The theory is not complex however. Modification of this method for special purposes will be explained later. The apportioning of the demand cost however presents a more difficult question.

Demand Cost.—If all loads had similar characteristics, *i.e.*, similar load curves, with the peak coming at the same time it is evident that the demand charge for each would be simply proportional to its peak load. This assumption is sometimes used in figuring energy cost but is obviously not correct except in cases where all loads are similar or nearly so, such as for a plant serving residence lighting only. On this assumption, the demand charge per kilowatt may be reduced to a figure per kilowatt-hour, inversely proportional to the load factor, and this added to the kilowatt-hour charge determines the total cost of energy. Where both lighting and commercial power or other loads are carried, however, it is evident that the demand

responsibility of each cannot be accurately obtained in this way as the peaks come at different times and the loads show different characteristics throughout the day. For example the power peak might come at the same time as the station peak at perhaps 2 P.M., whereas the lighting peak comes at 8 P.M., the lighting load at 2 P. M. being only 30 per cent of its peak.

Again the assumption might be made that the demand charge of any load is proportional to its demand at time of station peak. This might sound reasonable inasmuch as the demand cost of the station is figured on the basis of its peak load. The demand responsibility for any customer is sometimes computed on this basis by multiplying his total connected load by his demand factor to get his individual demand, dividing this in turn by the diversity factor on the line for his proportion of the line demand, this by the diversity factor of the substation and so on back to the generating station, thus determining that customer's proportion of the station peak. Of course a careful determination of diversity and demand factors is necessary for this. Here again the variable characteristics of the different loads make this assumption erroneous except in special cases.

Take for a simple example a small station of 1,000 kw. serving two customers A and B. A takes 1,000 kw. for 6 hr. each day. B takes 600 kw. for the remaining 18 hrs. but does not overlap A. On the assumption of demand proportional to individual peak load, A's cost would be $^{10}\!/_{16}$ and B's $^{6}\!/_{16}$ of the total demand cost. On the assumption of demand proportional to load at time of station peak, A's proportion would be the whole station demand and B's nothing (see Fig. 1).

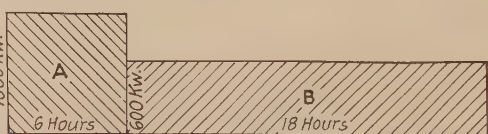

Fig. 1.

If, however, the station may be considered as consisting of two parts, one of a capacity of 600 kw. and one of 400 kw., it will be readily seen that the 600 may be assumed to operate 6 hr. for A and 18 for B while the 400 operates only 6 hr. for A (see Fig. 2). In this case then A's cost would be $\frac{400}{1,000} + \frac{6}{24} \times \frac{600}{1,000} =$

$\frac{2}{5} + \frac{3}{20} = \frac{11}{20}$ of the total, while B's would be $\frac{18}{24} \times \frac{600}{1,000} = \frac{9}{20}$ of the total. This theory can be extended to cover any number of loads with variously shaped curves. It may be stated in general that the cost of each unit (kilowatt) of the total demand should be divided in accordance with the length of time or number of hours it is in use, to obtain an accurate apportioning of demand cost. A full explanation of this theory with examples of its application may be found in "Central Station Rates in Theory

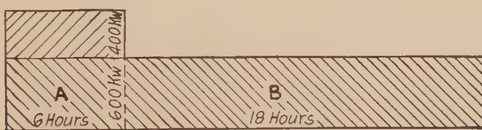

FIG. 2.

and Practice," by H. E. Eisenmenger, *Electrical Review*, Vol. 75, Aug. 2 and 9, 1919. Ordinarily sufficient data in regard to the various load curves may not be available or the degree of accuracy desired in the result will not warrant an extensive analysis on this basis. There is no doubt that theoretically it will give a more accurate distribution of the demand cost for most purposes than either of the other methods mentioned and the principles involved may often be used to advantage, even in a more approximate determination. For some special uses, as will be explained later, the second method given above is preferable.

General Method for Determining Demand Cost at any Point. In order to determine the demand cost at any point on the system other than the generating station the following general steps should be followed (see Fig. 3):

1. Apportion the generating-station demand cost among the various substations by one of the above methods, in accordance with their loads.

2. For any substation, add to its portion of the generating station cost the demand costs on its transmission cables and on the substation itself.

3. The total demand cost for the station can now be distributed among the various lines feeding out from it and by repeating the same method the cost at any point or for any customer may be determined. If more general figures only are desired the total demand cost for the station may be divided among various classes

of load such as, lighting, commercial power, street lighting, street railway, etc. By assuming average figures on line costs, the cost at any distance from the station for each class of load can be determined. The analysis may be extended in a similar manner

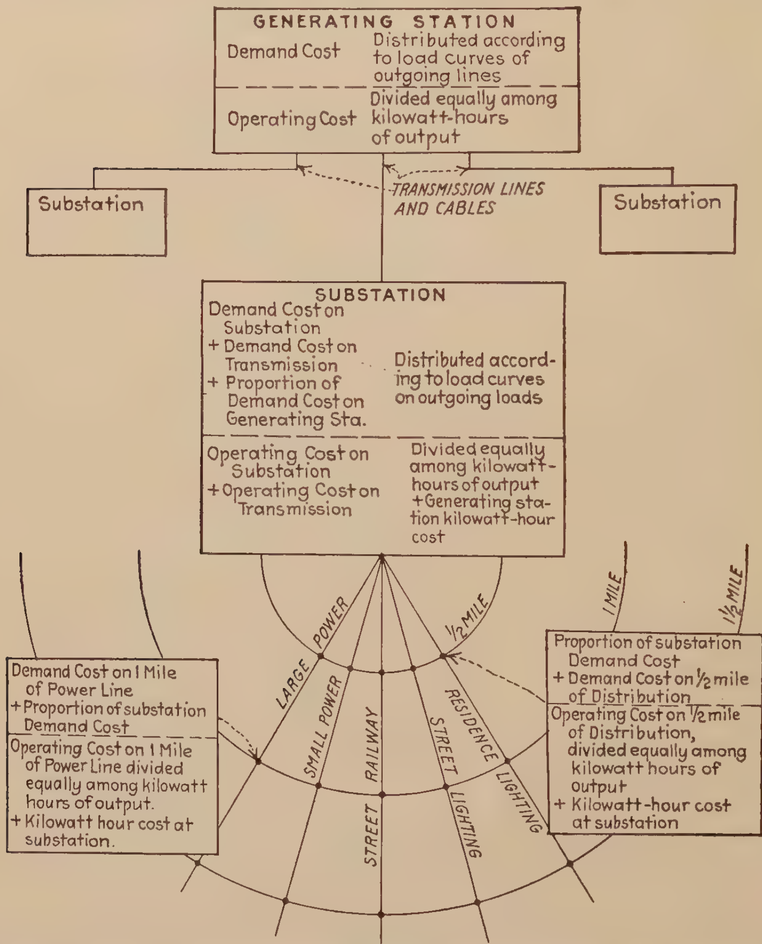

FIG. 3.—Diagram indicating general method of studying energy cost at various points on a system.

to cover suburban transmission lines, substations and distribution even where several substations intervene.

Any such analysis should give a fairly accurate demand charge at any point. It requires however quite a large amount of accurate data for its accomplishment. Detailed annual costs

on each subdivision of the system must be known. Also the load curves throughout the year, for each load considered such as substations, power lines, etc., must be studied to obtain one or more characteristic curves for each as a basis of analysis. Where a load shows an appreciable seasonal variation several curves may be necessary as for high load, low load and average load. In many cases it will be found practicable to determine a general characteristic curve for each type of load, such as residence lighting, street lighting, street railway, large power, small power, etc., and refer all loads of that type to it, assuming that the curve will be always proportional to the peak load. The collection, classification and proper selection of data will be found to be a large part of the whole problem of determination of energy cost and requires the application of a high degree of engineering knowledge and judgment.

In order to make such a detailed analysis as proposed above the following information should be available.

1. *Cost of Generating Station.*
   (*a*) First cost and present value of various parts in sufficient detail so that demand, energy and customer's costs can be separated.
   (*b*) The proper percentages of interest, taxes, depreciation, etc., chargeable to each part.
   (*c*) Operating costs in some detail.
   (*d*) Determination of proper percentage of each item of fixed and operating charges belonging to each of the three divisions of cost.
2. *Cost of Transmission-line Cables.*
   (*a*) First cost and present value of cable lines to each substation with proportional cost of tie lines between stations.
   (*b*) As above.
   (*c*) Such operating cost as may be chargeable to cables, repairs, etc.
   (*d*) As above.
3. *Cost of Substations.*
   (*a*) First cost and present value of each substation in sufficient detail.
   (*b, c, d*) As above.
4. *Cost of Overhead Lines.*
   (*a*) First cost and present value of average line of each class, transmission, power line, circuit, direct-current feeder, railway feeder, and secondary distribution including transformers, per unit length.
   (*b, c, d*) As above.
5. *Characteristic Curve for Generating Station.*
   (May be taken for seasons or months instead of for year.)
6. *Characteristic Curves for each Substation.*
   (Corresponding with generating-station curves.)
7. *Characteristic Curves for Each Class of Load out of Each Substation.*
   (Or for each line.)

Total Charge Per Kilowatt-hour.—All the formulas which will be developed later include the cost of energy as one total charge per kilowatt-hour. The computations might be made with the demand charge and the kilowatt-hour charge as two separate quantities, but it is found more convenient to use a single charge per kilowatt-hour. If the above analysis is carried out in detail the demand charge for energy will be determined as a different amount for each type of load and for any distance from each substation. The kilowatt-hour charge will be the same for all loads in any one locality. Since the average load factor for any type of load considered may be determined, the demand charge can be reduced thereby to a charge per kilowatt-hour, since kilowatt hours = kilowatts $\times$ load factor $\times$ 24 $\times$ 365 per year. This added to the output or kilowatt-hour charge will give the total charge per kilowatt-hour for that class of load at that point.

Further Variations in Energy Cost.—Up to this point there has been considered, for any type of load and locality, only the average unit cost of the total energy delivered, without attempting to differentiate between costs for large loads and small loads of the same type, high power factors and low power factors, losses and used power, etc. There will now be indicated some of the possible variations in this cost and methods of studying them will be suggested.

It may be shown that all units of energy even of the same class and locality do not cost the same. For example, energy lost has a higher unit cost than energy used. Again, not only does the total kilovolt-amperes of any load increase as power factor decreases but that increase costs more *per unit* than the average cost if a 100 per cent power factor were obtained. Further, under some conditions an increase in a load will cost more per unit than the former average.

The underlying theory involved may be readily seen if we consider the cost of lost or waste energy for example. Most substations are regulated in some way, and if the energy losses between the substation and the customer could be reduced the current in the transmission cable to the substation would be also reduced. Since losses are proportional to the square of the current, the cable losses eliminated by this reduction of the load on the cable would be proportional to the difference in the squares of the currents before and after reduction. Hence it will be

seen that *reduction in loss* per unit *reduction in load* is greater than the average loss per unit of the total load, after reduction. In other words, the loss in the cable due to the upper part of the load, which may be eliminated and hence may be considered waste, is greater per unit than the loss due to useful load. Since this energy must be supplied at the generating plant, the energy generated will be more per unit for this waste energy than for the remainder of the load or useful load and the cost will hence be more. This may be shown mathematically as follows:

Suppose a substation B is supplied through cables from a generation station A—

(A) ———————————————————————————— (B)

W = load in watts at B,
P = percentage of W lost beyond B, which may be considered waste power which might be conserved in some way,
$p = P/100$.
E = Voltage at B,
R = Resistance of cable.

For simplicity assume single-phase cable, unity power factor. Then

I^2R losses in AB due to total load $W = 2\left(\frac{W}{E}\right)^2 R$,

I^2R losses in AB due to power load only $= 2\left(\frac{(1-p)W}{E}\right)^2 R$,

I^2R losses in AB due to losses beyond $B(=pW)$ =

$$2\left(\frac{W}{E}\right)^2 R - \left(\frac{W}{E}\right)^2 R + \left(\frac{W}{E}\right)^2 R(2p - p^2),$$

or expressed in percentage of total loss in AB

$$= 100(2p - p^2) = 2P - \frac{P^2}{100} \text{ per cent} \qquad (1)$$

If the loss or waste power at B is a comparatively small percentage of W it will be seen that the losses in AB due to that waste are nearly twice that percentage of the total losses. (Ten per cent waste at B means that 19 per cent of losses in AB are due to that waste.)

If Q = percentage of W lost between A and B,

$$\frac{QW}{100} = \text{total loss in cable},$$

$$\frac{QW}{100}\left(2P - \frac{p^2}{100}\right)\frac{1}{100} = \text{loss in cable due to waste beyond } B.$$

But waste beyond $B = \frac{PW}{100}$.

Hence, the loss in the cable due to waste beyond B per unit waste =

$$\frac{\frac{QW}{100}\left(2P - \frac{p^2}{100}\right)1/100}{\frac{PW}{100}} = Q/100\left(2 - \frac{P}{100}\right) \tag{2}$$

Since the average total loss in cable per unit total load at $B = \frac{Q}{100}$. If cost of energy per kilowatt at $A = C_e$

$$\text{Average cost at } A \text{ of total energy at } B = \left(1 + \frac{Q}{100}\right) C_e \tag{3}$$

Average cost at A of energy waste beyond B

$$\left(1 + \frac{Q}{100}\left(2 - \frac{P}{100}\right)\right) C_e \tag{4}$$

In other words a loss of Q per cent in line AB increases the average cost of total energy delivered at B by Q per cent. But if P per cent of that total energy represents losses which may be assumed to be reducible or waste, the average cost of such losses is increased by $Q\left(2 - \frac{P}{100}\right)$ per cent. The difference in average unit total cost and average unit waste cost is then $\frac{Q}{100}\left(1 - \frac{P}{100}\right) C_e$, or

$$\frac{\frac{Q}{100}\left(1 - \frac{P}{100}\right)100}{1 + \frac{Q}{100}} = \frac{Q\left(1 - \frac{P}{100}\right)}{1 + \frac{Q}{100}} \text{ per cent of average cost}$$

at A of total amount of energy delivered at B. (5)

For example, if 1,000 kw. were transmitted over a line for 1 hr. with a loss of 6 per cent—if 10 per cent of that 1,000 represents reducible losses or energy wasted beyond the end of the line

$$P = 10 \qquad\qquad Q = 6$$

If energy at A costs .01 per kilowatt-hour the average cost per kilowatt-hour delivered at B including losses in AB but not fixed charges on line $= \left(1 + \frac{6}{100}\right) .01 = .0106$ per kilowatt hour (from Eq. 3).

The average cost of energy waste beyond B is

$$\left(1 + \frac{6}{100}\left(2 - \frac{10}{100}\right)\right) .01 = .01114 \text{ (from Eq. 4).}$$

The difference is .00054 which is 5.1 per cent of the average. The same percentage of increase would be obtained from Eq. 5.

$$\frac{6\left(1 - \frac{10}{100}\right)100}{100 + 6} = \frac{540}{106} = 5.1 \text{ per cent}$$

It must be noted that the above deals with the average cost at A of energy delivered at B. In order to determine the average cost at B of energy delivered at B the charges on the line AB must be included. These should be practically proportional to W and hence would average the same per unit for both useful load and for losses.

The above merely establishes the fact of the increased cost of waste energy and must not be considered as indicative of the actual amount of that increase. In determining this a number of variable quantities must be considered:

1. The demand charge for waste energy at any point should be figured by the second method given under that heading, *i.e.*, making up the demand charge, for any load, of charges proportional to the amount of that load at the time of peak load on station, transmission cable, substation, etc. The reason for this is seen when it is considered that the demand cost on the station, for example, is considered proportional to the station peak. If that peak can be reduced by eliminating waste energy (without reducing the useful load) the demand cost will be likewise reduced in proportion. This line of argument does not apply to useful load since, for this, the company is receiving a monetary return. The actual demand cost should be distributed to such loads by the more equitable third method. In other words, a reduction of losses at off-peak time would not affect the actual demand cost of the station nor the distribution of that cost among the useful loads. A reduction of useful load at off peak would reduce the revenue and hence increase the cost to other loads.

2. The example above uses the energy charge as a single total charge per kilowatt-hour and the loss at a definite percentage. Since the percentage loss varies as the load varies during the day the increase in cost will not depend on the percentage losses at maximum load. The variation in loss will affect both the demand charge and the kilowatt-hour charge and each in a different proportion.

3. If several loads of different sizes and characteristics are considered the problem is further complicated.

4. If lines are operated under the most economical load, the losses may be considered as useful load rather than waste.

As has been stated before, the theory here explained can also be applied to determination of the additional cost of low power factor, of load increases (in some cases) and other similar problems. It would appear that each unit of energy comprising any load might be conceived as having a different cost. In dealing with increases in useful or saleable load we have the further consideration of economical loading of lines so that it would not be correct to say that an increase in load always costs more per unit than the average unit cost before the increase. In dealing with losses, however, a large percentage of loss may be assumed to cost more per unit than a small percentage. This idea of increased cost of losses and waste energy is an important one to bear in mind in all economical studies, since usually such a study is fundamentally the establishing of the most economical relation between cost of energy loss and other costs.

This chapter is intended to be more a suggestion as to how the problem of cost of energy and energy losses may be studied rather than a complete solution or a recommended method to cover all cases. It may be easily seen that the study of energy cost might be carried to an almost infinite degree of refinement. Many economical studies are general for any part of a system and are intended to cover a period of time well into the future. For these cases a very detailed determination of energy cost would not seem necessary. If the more accurate costs are once determined however, they can easily be averaged for a more general problem. Lack of time and of the necessary data will, in many cases, limit the study to more or less approximate results. It is hoped, however, that the methods and principles here explained will give a good idea of the nature of the problem. Detailed analyses of energy cost, when data is available, will well repay the effort expended. Even a more approximate study with these theories in mind will indicate the relative costs of energy at various parts of the system, of various types of loads, and the relation between the cost of waste and useful energy.

CHAPTER V

LOAD CHARACTERISTICS

Power Factor—Balance Factor—Demand Factor—Diversity Factor—Load Factor—Equivalent Hours

The consideration of the cost of energy, in the previous chapter brought forth some of the quantities that are used in analyzing the character of a load and its relation to other loads. It was also indicated that there are many kinds of loads and that each one of these can be expressed in terms of its characteristics. It is proposed here to define and briefly comment on some of these terms: power factor, balance factor, load factor, demand factor, diversity factor, and on equivalent hours. When available the definition of each one of these terms will be taken from the "Standardization Rules" of the A. I. E. E.

Power Factor.—"The ratio of the power (cyclic average as defined in No. 26) to the volt-amperes. In the case of sinusoidal current and voltage, the power factor is equal to the cosine of their difference in phase." (Power in an alternating-current circuit. The average value of the products of the coincident instantaneous values of the current and voltage for a complete cycle, as indicated by a wattmeter.)

Since the date of this issue of the rules there has been considerable discussion as to the best definition for power factor. However for the purpose of this book the above general definition is sufficient. From the point of view of the designer of an economical line the power factor is usually determined in advance by the load at the end of the line and it is not within his province to change its value. The interesting point to him is really the kilovolt-amperes his line has to carry, as it is the current of a load that determines, on account of the energy losses, not only the economical loading of an old line, but the economical size of wire for a new line destined to carry a given known load. It is evident, however, that while making studies in economical handling of loads the effects of power factor will be most forcibly brought to view and that the desirability of high power factor and general power-factor improvement, particularly on lines carrying large amounts of power loads, will be shown to be most imperative for increased economy.

Balance Factor.—While balance factor has never been positively defined it should be a means of expressing the divergence between an unbalanced load on a polyphase circuit and the same load when perfectly balanced. While it has been assumed throughout this book that loads were balanced, it is evident that many problems will arise where it will be necessary when solving for economical design, to make allowance for such unbalance. Large single-phase loads on polyphase circuits will often result in this necessity.

Demand Factor.—"The ratio of the maximum demand of any system or part of a system, to the total connected load of the system, or of the part of system under consideration."

(The demand of an Installation or System is the load which is drawn from the source of supply at the receiving terminals averaged over a suitable and specified interval of time. Demand is expressed in kilowatts, kilovolt-amperes, amperes, or other suitable units.)

(The Maximum Demand of an Installation or System is the greatest of all the demands which have occurred during a given period. It is determined by measurement, according to specifications, over a prescribed time interval.)

Demand factor is therefore the expression of the relation between apparent load or connected load and the largest actual load that will be expected at any time on an installation. For example, if a house is wired for 30 outlets each using a 40-watt lamp and the greatest number of these operating at one time is nine, the demand factor is $\frac{9}{30} = .3$ or 30 per cent. On the other, hand if a service is wired for a range of 5.5-kw. capacity and if at any time this range is operated with all the elements turned on the demand factory becomes unity.

Diversity Factor.—"The ratio of the sum of the maximum-power demands of the subdivisions of any system or parts of a system to the maximum demand of the whole system or of the part of the system under consideration, measured at the point of supply."

Here we express a load relation between various loads of the same or of different demand factors and other characteristics. For instance, if we take 10 houses each having the 30 outlets as above and the same demand factor of .3, the total demand will not be $\frac{10 \times .3 \times 30 \times 40}{1000}$ or 3.6 kw. but some smaller amount

due to the fact that the maximum demand of all the houses are not simultaneous. Therefore if the maximum load of these houses taken together is 1.8 kw. the diversity factor is $\frac{3.6}{1.8} = 2.$

Also taking several electric ranges as above the diversity factor may be .3 while each range at some time or another will be operating at a demand factor of unity.

Similarly the diversity factor between transformers, between substations, between lines can be obtained.

Load Factor.—"The load factor of a machine, plant or system. The ratio of the average power to the maximum power during a certain period of time. The average power is taken over a certain period of time, such as a day, a month, or a year, and the maximum is taken as the average over a short interval of the maximum load within that period.

"In each case, the interval of maximum load and the period over which the average is taken should be definitely specified, such as a 'half-hour monthly' load factor. The proper interval and period are usually dependent upon local conditions and upon the purpose for which the load factor is to be used."

Since in economical design annual costs are generally the basis of analysis it is evident that the period of time to be used will be 1 year. In this book equivalent hours have been used for determining energy losses over a line as shown below.

Equivalent Hours.—It is usually convenient in studying a given type of load, such as residence lighting for example, to obtain the energy losses per year in terms of the load carried. By "load carried," the peak load for the year will be meant, since it is for that load that the size of the wire and transformers must be determined. Since the energy loss over a line is dependent on the square of the current at any time it is evident that the total loss is not proportional to the load factor since the load factor is determined from the average load and hence involves only the first power of the load at any time. The computation of total energy loss in terms of peak load must be based on the square of the loads at any time. It is therefore convenient for this purpose to determine for each of the various classes of load considered a quantity which has been called the "equivalent hours."

"Equivalent hours" may be defined as "the average number of hours per day which it would be necessary for the peak load

of the year to continue in order to give the same total energy loss as that actually given by the variable load throughout the year." It is a quantity which, if multiplied by the loss at peak load on any line gives the average loss per day over the year. If this average daily loss is then multiplied by 365 the total yearly loss in kilowatt-hours is obtained. This times the cost of energy per kilowatt-hour gives the annual cost of energy loss. It is evident that, if the equivalent hours for any load and the peak demand of the year are known, the total yearly cost for losses would be $I^2R \times t \times 365 \times C_e$.

where I is the current at peak load,
R is the resistance of the circuit,
t is the equivalent hours,
C_e is the cost of energy per kilowatt-hour.

The quantity, equivalent hours, is also of use in determining the cost of energy per kilowatt-hour previously discussed since the cost of energy losses is a component of that cost.

If the characteristic curve for any type of load were available for a whole year, the sum of the squares of the current for each hour taken from that curve times the resistance of the conductor, would give the total yearly loss in watt hours. This however would be a tedious computation and in most cases impracticable. It is usually sufficiently accurate to obtain characteristic curves

TABLE 1

Circuit, number	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	190	180	180	175	160	135	120	125	160	175	180	195
2	200	180	175	165	145	140	110	130	200	195	205	210
3	175	165	150	140	120	125	90	115	160	165	180	185
4	185	175	180	150	140	140	105	130	175	185	200	210
5	155	155	155	150	135	130	100	130	160	175	180	195
6	150	140	135	125	110	100	90	80	120	135	150	150
7	245	225	210	200	190	180	120	140	225	245	285	285
8	235	225	225	210	200	185	150	205	250	280	280	280
9	225	225	215	225	175	175	135	170	215	235	240	250
10	185	185	185	175	150	140	130	140	170	185	225	240
11	125	120	115	110	105	105	90	100	120	125	135	150
12	130	120	110	105	100	100	85	100	125	135	145	130
Total......	2,200	2,095	2,035	1,930	1,730	1,645	1,325	1,565	2,080	2,235	2,405	2,480
Average of the 12....	183	175	170	161	144	137	111	130	173	186	200	207

for each month or at least a typical characteristic curve applicable to any month with allowance for the variations in the peak from month to month. An example of an actual calculation of equivalent hours on residence lighting circuits will indicate a method which can be followed in such computations.

The characteristic variation of the load on a residence-lighting circuit from month to month was obtained from Table 1 of maximum-current readings for each month during 1 year on 12 typical circuits in various districts and with various loads.

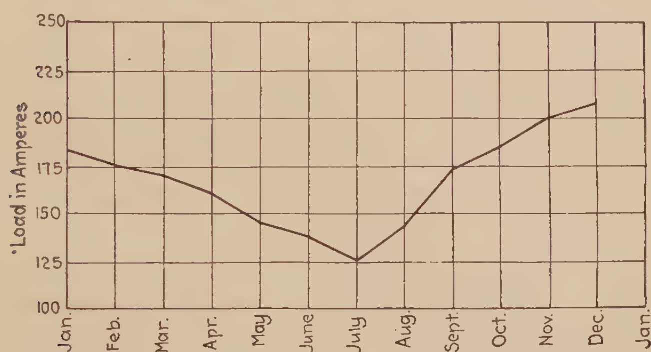

FIG. 4.—Average of monthly maximum loads on twelve typical lighting circuits.

The curve (Fig. 4) plotted from the above average values indicates clearly the variation of the monthly peak loads on a typical circuit. Since the peak for the year occurs in December, the peak for any month may be expressed as a fraction of this yearly peak as follows:

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
0.884	0.845	0.820	0.778	0.695	0.662	0.536	0.628	0.835	0.898	0.965	1.00

Hence, for example, the maximum current for February equals .845 multiplied by the maximum current for the year, etc.

For convenience the I^2R loss for any month may be considered equal to the loss for a typical day in the month times the number of days in the month, since the variation of load from one month to the next is not enough to warrant more detailed computation.

The I^2R loss in a feeder for any one day would be very nearly equal to the sum of the squares of the current readings for each

hour during the day multiplied by R. For a great part of the day, however, the load on a lighting circuit is very light, the heavy load and hence most of the loss occurring within a few hours in the evening. By adding the squares of the hourly current readings throughout a day and dividing by the square of the maximum current for the day a figure is obtained which represents the number of hours for which the peak load for the day would have to be carried steadily to produce the same I^2R loss.

For a strictly accurate calculation, enough data should be available to determine the shape of the load curve on a typical circuit for one typical day for each month in the year. If this is not possible, however, a fairly accurate approximation may be arrived at if the load curves for only one or two months are available. If the equivalent number of hours at peak load for a typical day for these months is determined the equivalent hours for the other months may be calculated more or less accurately by making them proportional to the number of hours between sunset and about 10 P. M. allowing a little additional time in the winter months for the morning lighting peak. The figures obtained in the present case for these monthly equivalent hours at peak load will be found in the first column of the accompanying table.

The loss for any day in a month is proportional to the square of the maximum current for the day multiplied by the equivalent hours per day at peak load as obtained above. If now we assume that the average daily peak will be 95 per cent of the peak for the month, the loss for this day is proportional to the square of the maximum current for the month multiplied by $.95^2$ multiplied by the equivalent hours per day at daily peak load as determined above. The monthly peak however is equal to a certain fraction of the yearly peak as shown before. Hence the loss for this day in terms of the yearly peak is proportional to the maximum current for the year, squared, multiplied by this fraction for the month, squared, multiplied by $.95^2$, multiplied by the equivalent hours per day at daily peak load. If this figure is multiplied by the resistance of the circuit, R, the actual loss is obtained. For example, the equivalent hours per day in terms of daily peak load as determined for February are $4\frac{1}{2}$. The peak for February is .845 of the yearly peak. Hence for a typical day in February the $I^2\ R$ loss equals (yearly maximum current)$^2 \times .845^2 \times .95^2 \times 4.5 \times R$.

The total yearly loss would be the sum of the daily losses thus obtained. An average of the figures for each month would then give the average loss per day. Since the quantity (yearly maximum current)$^2 \times R$ is the loss due to the yearly peak load, the average number of hours per day which that load must continue may be obtained by averaging the other factors entering into the computation of average daily loss in terms of yearly peak. These are the quantities (equivalent hours per day at daily peak load) $\times \dfrac{(\text{monthly peak})^2}{(\text{yearly peak})} \times 95^2$ as determined for each month. The resulting average is the value of "equivalent hours" for that load.

The following table shows the data worked out from the above example of residence lighting.

TABLE 2

Month	1 Equivalent hours at peak load	2 (Peak for month) / (Peak for year)	3 (Monthly Peak)^2 / (Yearly Peak)	1 × 3
January	5¼	.884	.781	4.10
February	4½	.845	.714	3.21
March	3¼	.820	.672	2.185
April	2¾	.778	.606	1.67
May	2¼	.695	.483	1.088
June	1¾	.662	.438	.767
July	2	.536	.287	.574
August	2½	.628	.395	.987
September	3¼	.835	.697	2.263
October	4	.898	.806	3.224
November	5	.965	.931	4.655
December	5½	1.00	1.00	5.500
Total	....	......	.....	30.223
Average	....	....	....	2.519

$2.519 \times (.95)^2 = 2.27$ eq. hr. per day at peak load.

The above indicates that for the example used of purely light-

ing load the total loss for the year will be the same as if the peak load were carried 2.27 hr. per day throughout the year.

Corrections for Special Conditions.—The above figures are subject to correction under certain conditions. Actual loads on circuits were used and no allowance was made for the normal increase which might be expected on lighting load due to additional customers, the increased use of current by old customers, etc. This might be a satisfactory figure for use in many cases but there might be occasions in which a load showing only seasonal variations would be encountered. For example, if distribution transformers are kept loaded nearly to capacity, new installations would care for the yearly increase and each individual transformer would show nearly the same load from year to year. In case the annual rate of increase is known, a correction can be applied to the figures given for each month to reduce it to the same maximum load for the year. In the example given, the average yearly increase over a number of years was found to be 20.3 per cent or 1.69 per cent per month. Assuming December as the yearly peak, the ratio of the peak for each month to the yearly peak was corrected by a proportional part of the yearly increase, *i.e.*, for November it was increased by 1.69 per cent, for October 3.38 per cent, etc.

TABLE 3

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Monthly ratio[1]	.884	.845	.820	.778	.695	.662	.536	.628	.835	.898	.965	1.00
Correction factor	1.177	1.160	1.143	1.126	1.119	1.102	1.085	1.068	1.051	1.034	1.017	1.00
Corrected ratio	1.040	.981	.937	.876	.778	.729	.581	.571	.878	.930	.980	1.00

[1] Monthly Ratio $= \frac{\text{peak load for month}}{\text{peak load for year}}$ uncorrected.

The above indicates that the actual corrected peak on such load is in January but not enough difference will be introduced to necessitate a revision of the figures to that basis.

If now the equivalent hours are computed on the basis of the above ratios a new figure is obtained for a loading with no yearly increase. In this case the equivalent hours thus corrected are computed to be 2.65 instead of 2.27 as determined for actual loading, with a yearly increase.

A further correction may be applied if, as in case of transformers, the full-load capacity is to be used in studying losses rather

than the actual load carried. If, for example, on the above circuits the transformers were carrying, on an average, 89 per cent of their full-load capacity, the equivalent hours based on connected capacity would be $2.65 \times .89^2 = 2.07$. That is to say, the year's loss in energy would be equal to the full load current on the transformer carried 2.07 hr. per day throughout the year.

Other corrections may be necessary to meet particular conditions. The above will give an indication of how such corrections should be applied.

The matter of equivalent hours should be carefully studied and as accurate figures as possible obtained for various classes of load. The values will vary, naturally, for different sections of the country as well as for different localities in the same section or on the same system. The habits of a community, as to hours of rising and retiring, etc., will affect the value for lighting loads. On power loads, of course, the nature of the industry will be a controlling factor. In general for the cases coming within the experience of the writers the values lie within the range given below:

Power load—from 0 to 10 eq. hr.
Residence lighting—from 2 to 3 eq. hr.
Store lighting (small)—from 2 to 3 eq. hr.
Store lighting (large)—from 2 to 5 eq. hr.
Street lighting—from 5 to 10 eq. hr.

Relation between Load Factor and Equivalent Hours.—It is interesting to find what the relation between load factor and equivalent hours is, especially as one or the other quantity may be available in a problem, while the other is necessary for the solution at hand. For our purpose it will be found particularly useful in making an approximate determination of equivalent hours if the load factor is known.

Limits may be established within which the value of the relation between load factor and equivalent hours will lie in all cases.

The extreme cases are as follows:

I. The peak load is on for a short time only. The remainder of the load curve is flat for the rest of the day. See Fig. 5.

In this case the amount of the continuous load divided by the momentary peak gives the load factor.

The I^2R loss $= I_{av.}^2R \times 24 = (LF \times I_{max})^2 R \times 24.$

Equivalent hours at peak load $= \dfrac{(LF)^2 \times I_{max}^2R \times 24}{I_{max}^2R} =$

$(LF)^2 \times 24.$

II. The peak load is continuous for a part of the day—the load is 0 thereafter. See Fig. 6.

In this case the load factor is the number of hours the load is on, divided by 24.

FIG. 5.

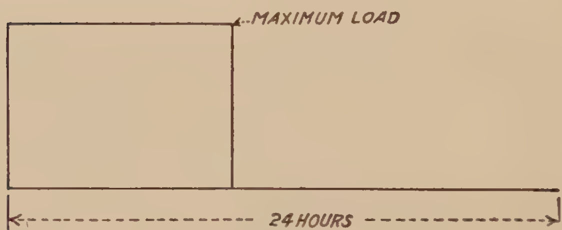

FIG. 6.

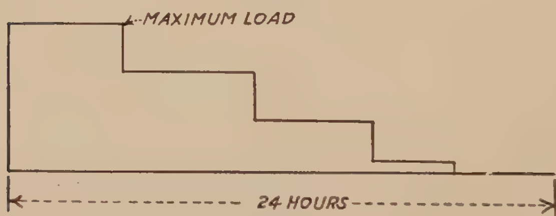

FIG. 7.

The I^2R loss $= I^2_{max}R \times (LF \times 24).$

Equivalent hours $= LF \times 24.$

III. For any intermediate arrangement of load. See Fig. 7.

At any time, t_n, let the value of $I_n = p_nI_{max}.$

The load factor $= \displaystyle\int \frac{p_nI_{max}dt}{I_{max}} = p_ndt.$

I^2R loss $= \int (p_nI_{max})^2Rdt.$

Equivalent hours $= \int p_n^2dt.$ (6)

It is evident from the nature of the curves that for any load factor Cases I and II are limiting cases since no greater distribution of load can be obtained than Case I and no greater concentration than Case II. The rule can be stated as follows:

"For any given load factor, the corresponding value of equivalent hours will be somewhere between the limits of (load factor $\times$ 24) and (load factor)$^2 \times 24$.

CHAPTER VI

GENERAL EQUATION

Kelvin's Law—General Method of Solving Problems—Presentation of Results

The previous chapters have dealt largely with the data necessary for the economical study of distribution problems, and the methods of obtaining that data. The cost of material and labor, the annual charges on these items, the unit cost of energy for different loads and the annual cost of energy losses have all been taken up in some detail. Once the data is collected, there still remains the problem of so utilizing it as to obtain the most economical conditions for the line or lines under consideration. Also, means must be found for so exhibiting the results, by equations, graphs, tables, etc., that they will be convenient of application to present problems and to future similar problems and subject to easy revision with changing prices.

Although every problem of this nature that is considered will present certain characteristics of its own which make it different from all others, there are certain underlying principles and methods of procedure which are applicable to all. A brief discussion of these will be given here. A good understanding of these general methods will simplify the study of their application to particular problems, which will follow in subsequent chapters.

Kelvin's Law.—To Sir William Thompson (Lord Kelvin) is generally attributed the basic study of economical conduction of electrical currents. In 1881 he expressed the principle that "the most economical size of copper conductor for the transmission of electrical energy would be found by comparing the annual interest on the money value of the copper with the money value of the energy lost in it annually in the heat generated in it by the electric current . . . Contrary to a very prevalent impression and belief, the gage to be chosen for the conductor does not depend on the length of it through which the energy is to be transmitted. It depends solely on the strength of the current to be used supposing the cost of the metal and of a unit of energy

to be determined." In expressing this mathematically, the total annual cost was expressed as the sum of the fixed charges on the conductor and the cost of energy loss. The size of wire for which this would be a minimum was then determined, being that size for which the two component charges are equal. What is generally known as Kelvin's Law has been formulated from this, *i.e.*, that the most economical size of conductor is that for which the annual charge on the investment is equal to the annual cost of energy loss. Under modern conditions with the use of alternating currents, wide range of voltages, large variety of wire sizes, with and without insulation, etc., an indiscriminate use of Kelvin's Law as thus stated is liable to lead to considerable error. It will apply strictly only to problems for which the cost of conductor supports can be neglected (or is directly proportional to the size of wire), when the cost of any size of wire is directly proportional to the cross-sectional area of the copper, when no transformers, condensers, or other equipment need be considered and when the cost of energy loss is inversely proportional to the wire size. Needless to say, few problems could be included under the above. In most cases it is necessary to revert to Kelvin's original method which was to determine an expression for the total annual cost and from this to determine the most economical condition desired. This will include not only the investigation of the most economical wire size but also of the most economical voltage, the most economical route, the most economical size and spacing of transformers, and other similar questions. From this can likewise be determined the actual advantage, in dollars, of one installation over another, where it would be economical to change from one type of installation to another, and a great many other extremely useful considerations.

General Equation.—A general expression for total annual cost which will be applicable to most of the problems in electrical distribution lines can be set down. Naturally each of the items included will be somewhat different for different problems and all problems will not include all of these items. The symbol g will be used throughout to indicate percentage of fixed charges on investment. It will vary, of course, with different kinds of property. In this general equation, g will be used as a general symbol, *i.e.*, the expression ($g \times$ a quantity) indicates that the annual charges rather than the first cost are considered. Then we have:

Total annual cost = g (cost of right-of-way)
+ g (cost of poles and fixtures or underground ducts in place)
+ g (cost of conductors in place)
+ g (cost of transformers and transformer equipment installed)
+ g (cost of any special equipment used)
+ cost of maintenance, inspection, testing, etc.
+ cost of annual energy loss on line
+ cost of annual energy loss on transformers and other equipment. (7)

Individual cases may produce other charges which must be added but the above is characteristic.

Units.—In some specific problems, such as the comparison of economy between two sizes of conductor for some particular location and definite load, actual values for voltage, resistance, costs of materials and energy and other constants could be introduced at once into this equation and the total annual cost of each installation determined in dollars. In the usual case, however, it is desirable to make the study more general, covering a more or less wide variation in conditions, so that it can be utilized to reduce computation on future problems or in the determination of standards for a given class of installations. For this purpose it is advisable to represent as many of the quantities as possible by symbols and to carry these symbols through the computations as far as possible. This also facilitates revision of the formulas, graphs, etc., if a change in prices makes this advisable. For example, the following are some of the symbols most commonly used in this book:

TABLE 4

W = load in watts	ρ = resistivity of conductor material
$kw.$ = load in kilowatts	$\cos\theta$ = power factor
E = voltage	A = cross-sectional area of conductor.
I = amperes	W = weight of conductor per unit length
R = resistance of circuit	T = transformer size in kilovolt-amperes
X = inductance of circuit	
r = unit resistance of conductor	
x = unit inductance of conductor	

TABLE 4 (*Continued*)

R_t = equivalent resistance of transformers	g = per cent fixed charges, (interest taxes, depreciation, etc.)
C_r = cost of right-of-way per unit	t = equivalent hours
C_{cu} = cost of copper per pound	V = per cent voltage drop
C_{sr} = cost of stringing conductors	P = per cent power loss
C_e = cost of energy (subscript 1, 2, 3, etc., indicate variations)	

In any given case, most of these quantities will be fixed by the conditions of the problem and may be considered as constants so that the general equation for annual cost may usually be reduced, in its final form, to one containing only two or three variables, such as load in kilowatts or amperes, cross-sectional area of wire, power factor, or equivalent hours. Examples of this will be shown later.

Presentation of Results.—The equation for total annual cost once obtained, there still remains the question of getting from it the information desired in the best possible form for convenient use. It is found that there are, in general, three ways in which it is convenient to accomplish this. Conditions of the problem and the results desired will determine which one of these is most applicable in any case.

1. The actual annual cost in dollars may be plotted in a curve or series of curves. Where the equation for annual cost is expressed in more than one variable, such as size of wire and load, for example, one of these variables must be held constant in plotting any one curve. Sufficient number of such curves must then be plotted to show the required variation in that quantity. In this way a series of curves may be obtained, for example, one curve for each standard size of wire, showing the relation between annual cost and load on a line with that size. An example of this method is shown in Fig. 8. When the expression for annual cost contains more than two such variables, however, this method is usually not applicable.

2. In some cases with three variables entering into the total annual cost the following method will be found convenient. Suppose, for example, the annual cost depends on a variable value for equivalent hours, variable load and variable wire size. If the expression for annual cost for two standard sizes of wire are equated, the resulting equation plotted between load and equivalent hours shows the dividing line between economy for

one size or the other. The accompanying figure (Fig. 9) shows such a series of curves as determined for three-phase secondary

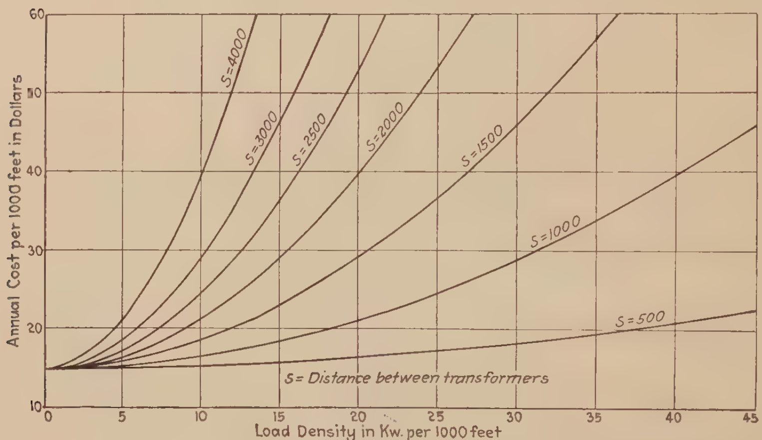

FIG. 8.—Annual cost per 1,000 ft. of 3 No. 4 secondaries. (Includes fixed charges on wire in place and energy loss.)

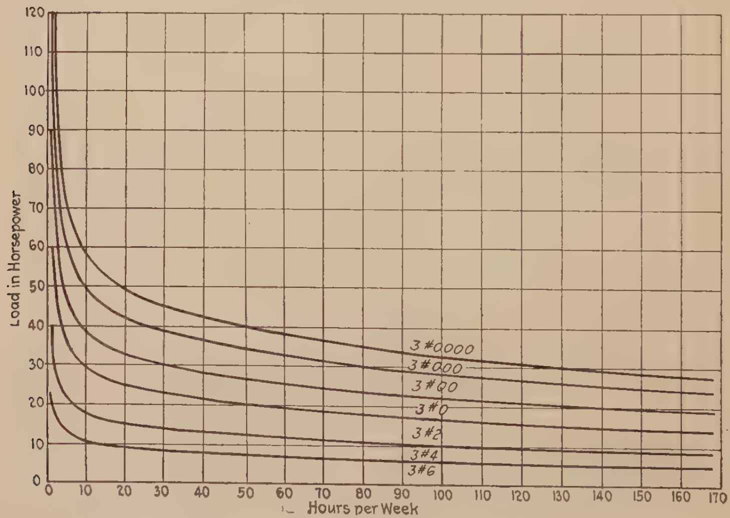

FIG. 9.—Most economical size of wire for three-phase secondaries (small power loads).

for a certain type of load. Any point lying between two curves indicates economy for the corresponding size of wire. This method has the disadvantage of not exhibiting quantitative

economy. The results are qualitative only, since the nearer the point lies to either limit of the area the less the relative difference between the cost with the size of wire indicated and the next adjacent size.

3. A third method gives results that are neither quantitative nor qualitative. Also, its application is usually somewhat limited. In certain cases, however, it is preferable to any other method. Where the expression for annual cost contains several variables, if these can all be reduced to terms of two variables

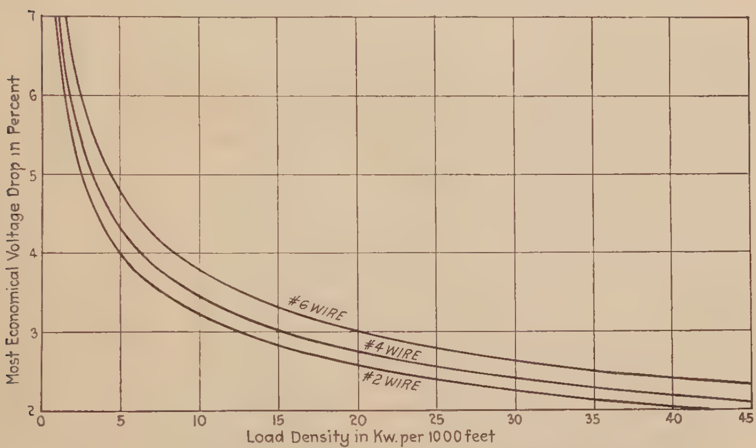

FIG. 10.—Most economical voltage drop on lighting secondaries.

(as percentage voltage drop and load, for example) and the first derivative of the resulting expression with respect to one of these variables be set equal to 0, the equation thus obtained will give the most economical relation between the two variables (as most economical voltage drop for any load under the given condition). The accompanying figure gives an example of a curve derived in this manner (Fig. 10).

As was suggested before, it will be found of considerable advantage to prepare the equations from which the final curves are plotted in as general terms as possible, *i.e.*, with as many of the constant quantities as possible represented by symbols. It is, then, a comparatively easy matter to revise the curves to meet changing prices of material and labor, or other conditions of load, voltage, power factor or materials of construction than those originally contemplated.

In some cases it may be found useful to develop the results in numerical tables or sometimes merely by a simple formula. Such cases are not the most usual, however, and the above methods will probably be found sufficient for most purposes. On some more extensive problems, all three methods of exhibiting data will be used.

In future chapters, a number of problems met with in practice will be taken up and their treatment in accordance with these methods will be described. Some of the details necessarily omitted in the previous discussion of these methods will be brought out in the individual problems and the procedure heretofore described in general terms will be actually carried through.

CHAPTER VII

POWER LOSS AND VOLTAGE DROP

Charts for Simplified Solution for Power Loss and Voltage Drop

Power Loss.—In most of the solutions of problems in economy appearing in this book, the power loss is introduced as a function of the load, the wire size, the equivalent hours, etc. The reason for this is evident from the nature of the methods used and the results desired. There are often cases, however, where the power loss alone is wanted. There will be given in this chapter simple curves, with their derivation, which enable the power loss to be quickly solved in the great majority of problems.

The most usual case of power loss is that due to the resistance of the conductor, *i.e.*, the I^2R loss. In high-tension transmission lines, leakage losses and corona losses become important. Charging current also has an effect on I^2R loss. Such problems, however, are comparatively rare in the work of most engineers and warrant special treatment when encountered. Methods of solving for corona loss, leakage, etc. are given in the handbooks and elsewhere and a discussion of them is beyond the province of this work. In the greater majority of problems, the I^2R loss is all that need be considered.

If D = length of line in feet,
W = the load at the receiver end in watts (= kilowatts $\times$ 1,000),
E = the receiver voltage,
A = cross-sectional area of conductor in circular mils,
ρ = resistivity of conductor material in ohms per mil foot,
$\cos\theta$ = the power factor of the load W.

$$I^2R\text{ loss} = \left(\frac{W}{E\cos\theta}\right)^2\frac{\rho D}{A}\times 2\text{ watts (for single-phase)} \qquad (8)$$

$$I^2R\text{ loss} = \left(\frac{W}{\sqrt{3}E\cos\theta}\right)^2\frac{\rho D}{A}\times 3\text{ watts (for three-phase)} \qquad (9)$$

If P = per cent power loss in terms of power delivered,

$$\frac{P}{100} = \frac{2W^2\rho D}{(E\cos\theta)^2A}\times\frac{1}{W}$$

$$P = \frac{200W\rho D}{(E\cos\theta)^2A}\text{(for single-phase)} \qquad (10)$$

$$\frac{P}{100} = \frac{W^2 \rho D}{(E \cos \theta)^2 A} \times \frac{1}{W}$$

$$P = \frac{100 W \rho D}{(E \cos \theta)^2 A} \text{ (for three-phase)} \quad (11)$$

For the same load, same voltage between conductors, and the same conductor size the loss with single-phase is twice that with three-phase.

If it is desired to consider the load, voltage and power factor at the source instead of at the receiver, if

W' = load at source in watts,
E' = voltage at source,
$\cos \theta'$ = power factor at source.

The line loss in percentage of load at source

$$P' = \frac{200 W' \rho D}{(E' \cos \theta')^2 A} \text{ (for single-phase)} \quad (12)$$

$$P' = \frac{100 W' \rho D}{(E' \cos \theta')^2 A} \text{ (for three-phase)} \quad (13)$$

For copper conductor $\rho = 10.8$ approximately. If load is expressed in kilowatts (kw), the formula becomes—

$$P = \frac{2.16 \times 10^6 \times kw. D}{(E \cos \theta)^2 A} \text{ (for single-phase)} \quad (14)$$

$$P = \frac{1.08 \times 10^6 \times kw. D}{(E \cos \theta)^2 A} \text{ (for three-phase)} \quad (15)$$

These formulas are comparatively easy to use. However it is believed the work may be somewhat simplified, especially where a large number of such computations are to be made, by use of the accompanying chart (Fig. 11). The use of this chart reduces the computation to a simple multiplication of round numbers.

The chart is plotted as follows:

A series of circular arcs (with the center at 0, 0) are drawn, each representing a given voltage. Voltages range from 0 to 150, but as will be shown below, the same arcs may be used for any voltage. Diagonal straight lines are drawn through 0, 0 at various slopes, each representing a given power factor. It is evident that the abscissa of the intersection of any arc with any diagonal gives the corresponding value of $E \cos \theta$.

For each standard wire size considered a curve is now plotted between $\frac{p}{kw. D/1{,}000}$ and $E \cos \theta$ giving for any voltage, power factor, and wire size the percentage power loss per kilowatt per

1,000 ft. Curves for other sizes of wire can be easily added if desired. The upper part of these curves was drawn to a condensed scale to give a greater range of values in the more unusual cases. The curves here shown are for three-phase. The

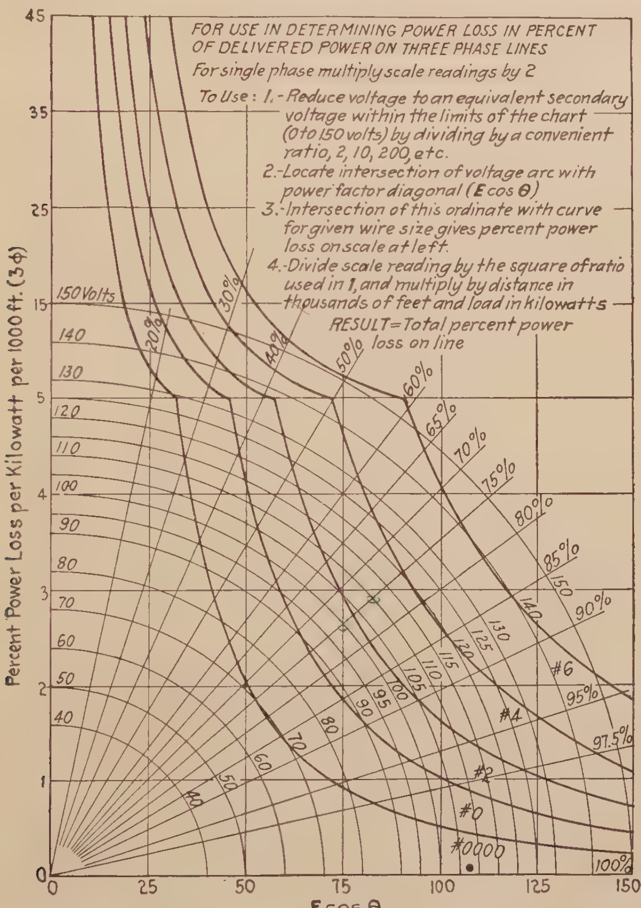

FIG. 11.—Power loss curves.

power loss for single-phase would be twice the values shown for three-phase.

The use of these curves is as follows:

1. Reduce the voltage considered to any equivalent secondary voltage within the scale of the chart (less than 150 volts) by dividing by some convenient transformation ratio such as 2, 4, 10, 20, 200, 400, 1,000, etc.

2. Locate intersection of voltage arc and power factor diagonal.

3. The intersection of the vertical through this point with the curve for the proper wire size gives the value of P on the scale at left.

4. Divide this scale reading by the square of the transformation ratio used, to give percentage power loss per kilowatt per 1,000 ft. for the given voltage. Multiply by the number of kilowatts and by the length of line in thousands of feet for total percentage power loss in terms of power delivered. (If line is single-phase multiply this quantity by 2.)

Take for example the following problem:

Load—1,200 kw. three-phase
Power factor—75 per cent
Voltage at receiver—4,400
Wire size—3 No. 0
Distance—8,000 ft.

$$\frac{4{,}400}{40} = 110 \text{ (transformation ratio } = 40)$$

$$\text{Power loss} = \frac{\overset{.38}{1.\cancel{5}\cancel{1}}}{\cancel{4}\cancel{0} \times \cancel{4}\cancel{0}} \times \overset{3}{\cancel{1}2\cancel{0}\cancel{0}} \text{ x } 8 = 9.12 \text{ per cent or } 109.4 \text{ kw.}$$

Voltage Drop.—In studying the economy of a line, it must not be forgotten that the element of good service is also important. Good service depends largely on good regulation which, in turn depends a great deal on the voltage drop of the line. Often, a line economically loaded will have too great a voltage drop. If it is artificially regulated, the cost of the regulator enters into the consideration of economy. In some cases the use of large conductors without a regulator may be more economical than smaller conductors with a regulator.

The computation of voltage drop is apt to be a tedious operation if carried out often. Several means of simplifying this have been published, such as the well-known Mershon Diagram and the Dwight chart.

When relating to high-tension lines, the problem of voltage drop involves consideration of line capacity, etc. These problems are usually of sufficient importance to warrant detailed computation. Formulas for this are given in the handbooks and elsewhere.

$$E_1 = E_2 \cosh \sqrt{ZY} + I_2 \sqrt{Z/Y} \sinh \sqrt{ZY}$$
$$I_1 = I_2 \cosh \sqrt{ZY} + (E_2/\sqrt{Z/Y}) \sinh \sqrt{ZY}$$

Where E_1 and E_2 are voltages from phase to neutral at the sending and receiving ends respectively; Z is the impedance per wire; Y is the admittance from phase wire to neutral.

For medium and low-voltage problems, it is usually sufficient

to consider inductive reactance and resistance only. The method and charts for computing voltage drop, given below, are based on these quantities. They are derived from the equation ordinarily used for such computations, *i.e.*

$$\frac{V}{100} = \frac{\sqrt{(E\cos\theta + \sqrt{3}RID)^2 + (E\sin\theta + \sqrt{3}\,X\,ID)^2} - E}{E}$$

(for three-phase lines)

$$= \sqrt{\left(\cos\theta + \frac{\sqrt{3}RID}{E}\right)^2 + \left(\sin\theta + \frac{\sqrt{3}\,X\,ID}{E}\right)^2} - 1 \quad (16)$$

Where V = per cent voltage drop in terms of receiver voltage,
R = resistance in ohms per foot,
X = inductive reactance in ohms per foot,
D = distance from source to receiver in feet,
I = current per wire.

But

$$P = \text{per cent power loss} = \frac{100\,W\,D}{(E\cos\theta)^2A} = \frac{100\sqrt{3}\,RID}{E\cos\theta}$$

(see above). (17)

Substituting

$$\frac{V}{100} = \sqrt{\left(\cos\theta + \frac{P}{100}\cos\theta\right)^2 + \left(\sin\theta + \frac{P}{100}\,\frac{X}{R}\cos\theta\right)} - 1$$

$$= \cos\theta\sqrt{\left(1 + \frac{P}{100}\right)^2 + \left(\tan\theta + \frac{P}{100}\,\frac{X}{R}\right)^2} - 1 \quad (18)$$

$$\frac{V}{P} = B = \frac{100\cos\theta}{P}\sqrt{\left(1 + \frac{P}{100}\right)^2 + \left(\tan\theta + \frac{P}{100}\,\frac{X}{R}\right)^2} - \frac{100}{P} \quad (19)$$

(The above computation is for three-phase but the resulting expression for B is the same for single-phase.)

B then is a quantity expressing the relation between per cent voltage drop and per cent power loss. It depends on the power factor ($\cos\theta$), size and spacing of conductors $\left(\frac{X}{R}\right)$ and per cent power loss (P), but is independent of load or voltage. It is evident that if B is known the per cent voltage drop may be easily obtained from the per cent power loss by the simple relation

$$V = BP \quad (20)$$

B may be plotted as shown in the accompanying curves (Fig. 12) (a) and (b). Since the variation of B with P is not great excepting for high-power factor and large conductors, three

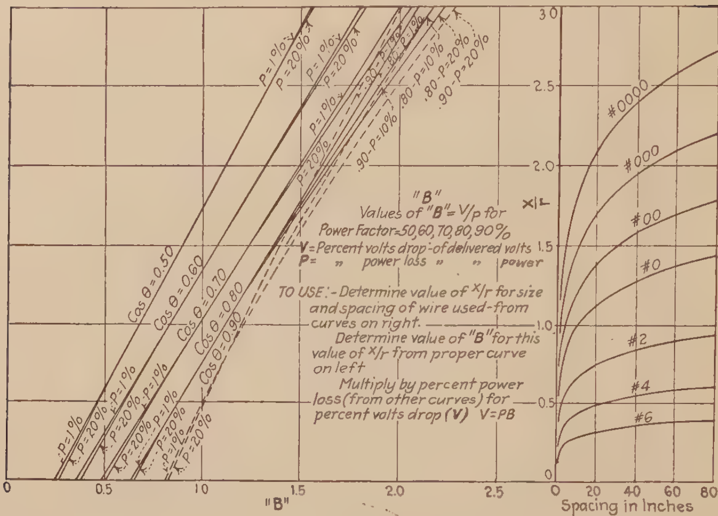

FIG. 12a.—Values of "B."

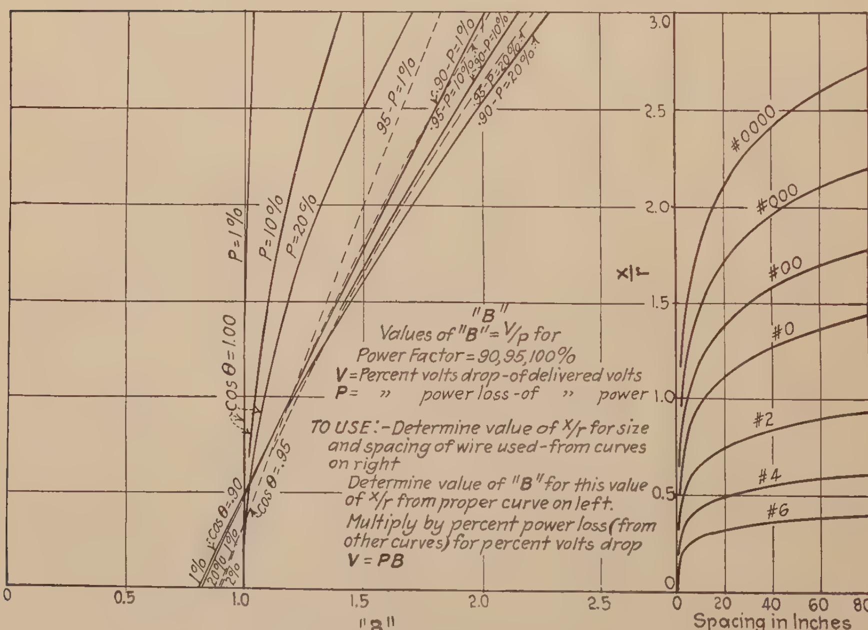

FIG. 12b.—Values of "B"

fixed values of P were chosen which cover the usual range of problems, *i.e.*, 1 per cent, 10 per cent and 20 per cent. For each power factor, a curve between B and X/R is plotted for each of these values of P. Other values of P may be interpolated

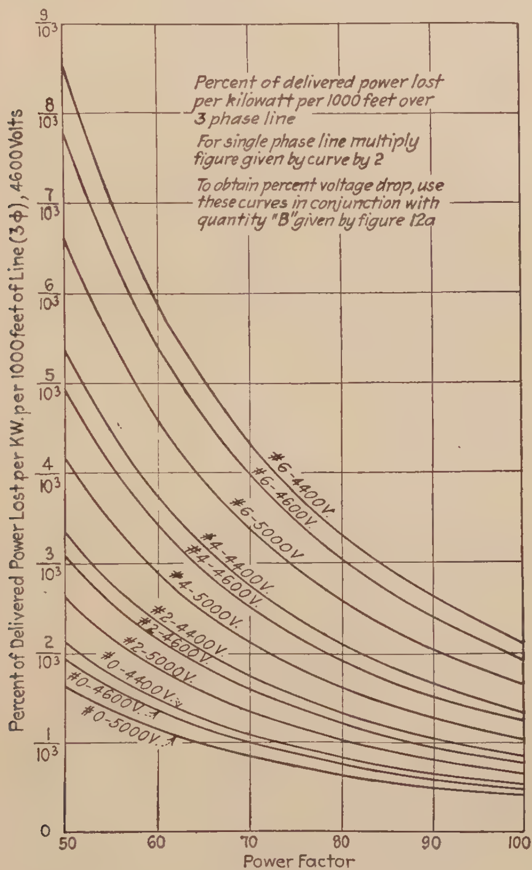

FIG. 13a.—Power loss curves.

if necessary. The value of X/R is shown directly by the curves on the right for standard sizes of conductor and any spacing.

Use of Curves.—The use of the curves is as follows: Locate the point on the X/R curve for the given wire size and spacing. (For three-phase unequal spacing use the equivalent spacing, $S = \sqrt{S_1S_2S_3}$). Pass across the sheet to the left on the horizontal through this point until it intersects the curve for the given power

factor and value of P. The corresponding value of B is found on the scale at the bottom. If this is multiplied by the value of P computed from Fig. 11, the desired value of V (per cent voltage drop) is obtained.

Example.—In the example given above for the determination of P, let the spacing between the wires be 28 in., 28 in., 56 in.

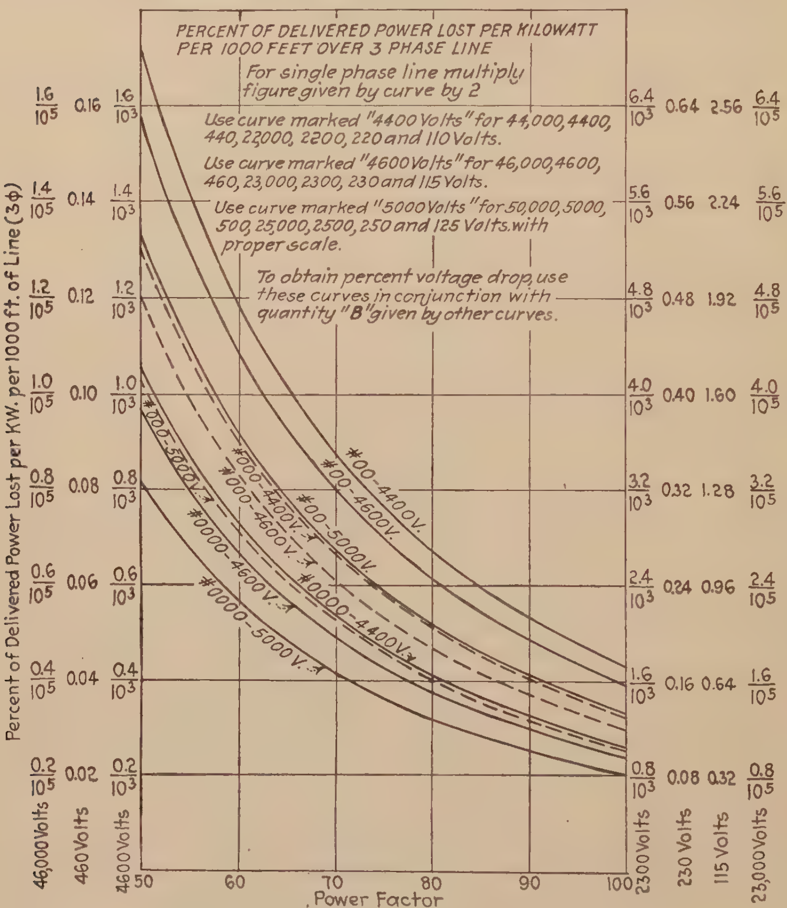

Fig. 13b.—Power loss curves.

The equivalent spacing $= \sqrt{28 \times 28 \times 56} = 35$ in.

Interpolating between the 70 per cent and 80 per cent power-factor curves

$$B = 1.18$$

$$V = 1.18 \times 9.12 = 10.75 \text{ per cent or } 473 \text{ volts.}$$

The curves given here are general and may be used for any voltage, load, length of line and spacing between conductors. The range of wire sizes and power factor is more limited but the curves could be easily extended to cover any desired value. For everyday use on the problems arising on any given system less general curves may be derived from these which still further simplify computations. As an example, Fig. 13 (*a*) and (*b*) shows a set of curves plotted for work on 4,600-volt lines. Three curves are shown for each wire size giving a range from 4,400 to

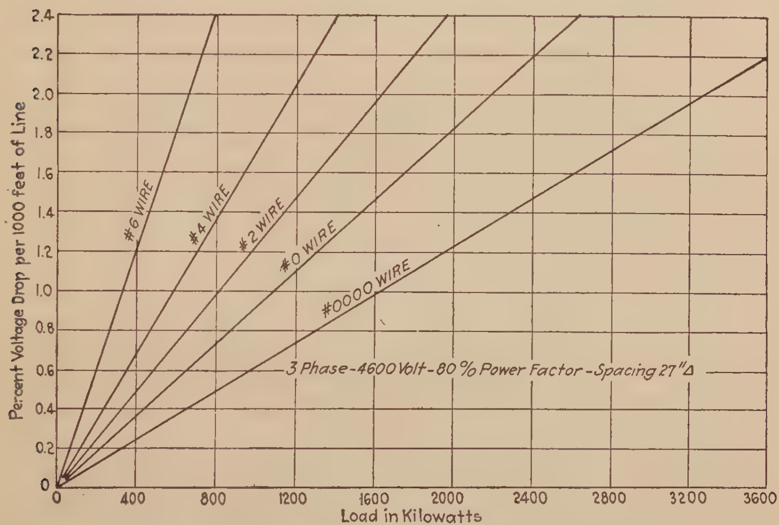

FIG. 14.—Curves showing per cent voltage drop (of delivered voltage) per 1,000 ft. of line.

5,000 volts. For any power factor (lower scale) the scale on the left gives the per cent power loss per kilowatt per 1,000 ft. Figure 13 (*b*) shows how scales for other voltages can be added.

Figure 14 shows another special curve giving per cent voltage drop per 1,000 ft. of line for 4,600 volts, three-phase, 80 per cent power factor, 27-in. spacing.

Figure 15, called "Load Curve for Power Lines", gives the distance to which any load can be carried on a three-phase, 4,600-volt line at 80 per cent power factor with a 10 per cent drop in voltage.

Figure 16 is a similar curve for single-phase lines with a power factor of 95 per cent.

Figure 17 is a load curve for 220-volt, three-phase secondary with 10 per cent drop.

Figure 18 is a load curve for $^{224}\!/_{112}$-volt, single-phase secondary with 3 per cent drop.

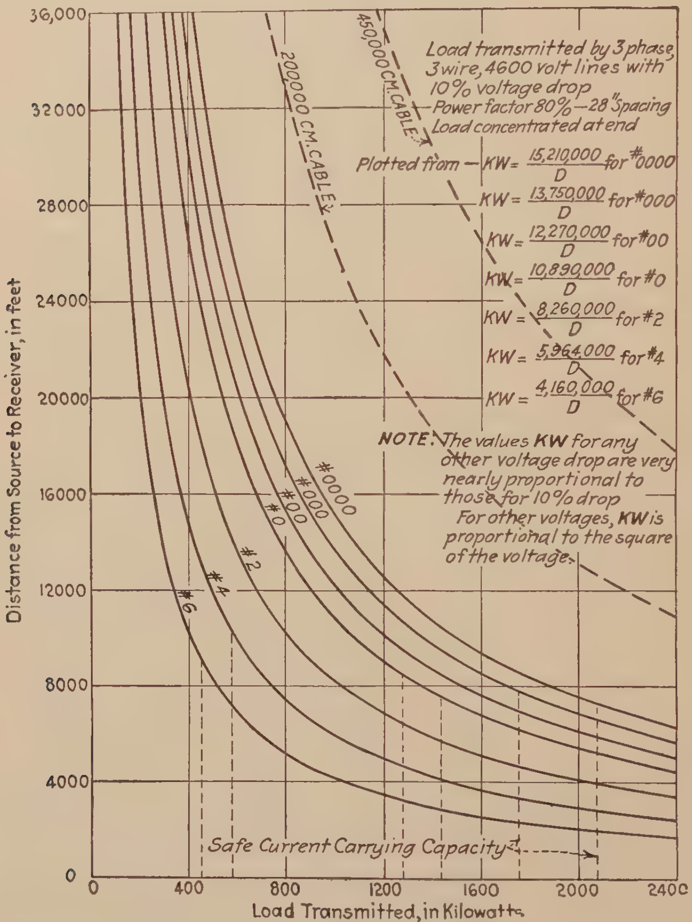

FIG. 15.—Load curves for power lines.

It is a comparatively simple matter to derive any such special curves desired by use of the general curves for power loss and B given above.

Approximate Method for Secondaries.—For low-voltage problems, such as for secondaries, an approximate determination

is usually as accurate as one more detailed. It will be found that the expression

$$\frac{V}{I} = \text{volts drop per ampere} = R \cos \theta + X \sin \theta \qquad (21)$$

while an approximation, is sufficiently accurate for most problems of this class.

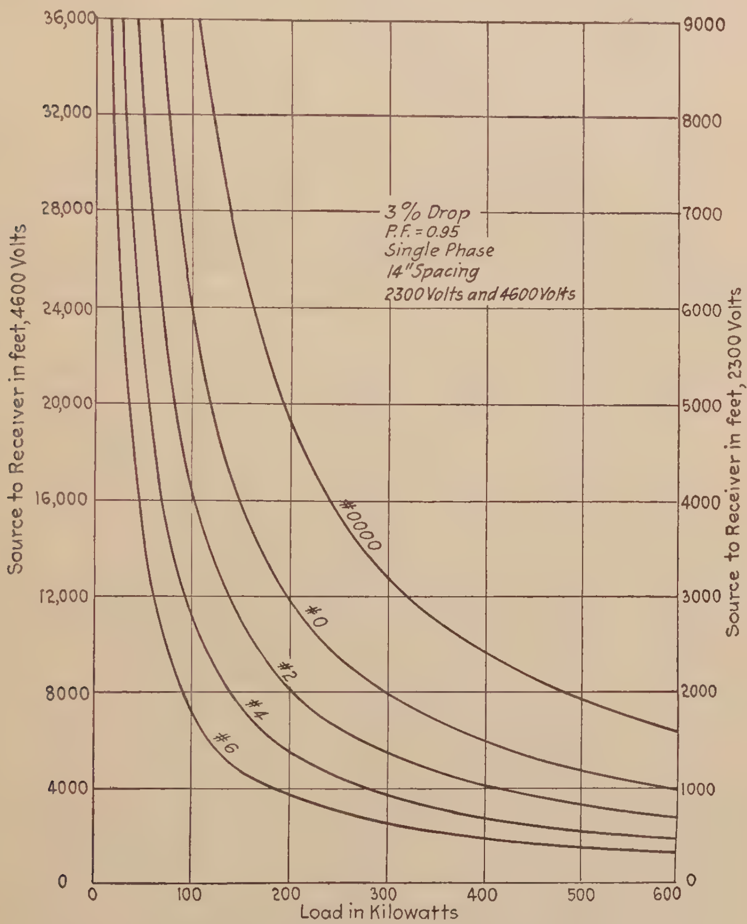

Fig. 16.—Load curves for single-phase lines.

As an example for the use of this expression, the curves in Fig. 19 (*a.b.c*) are given. The load considered is residence lighting on three-wire, $^{110}/_{220}$-volt secondary. The average per residence is assumed to range from 135 to 200 watts. Two different lengths

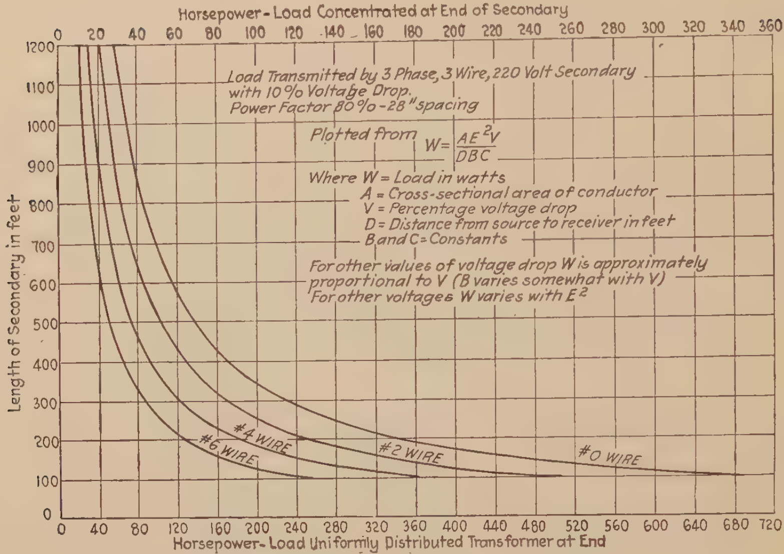

Fig. 17.—Load curves for power lines.

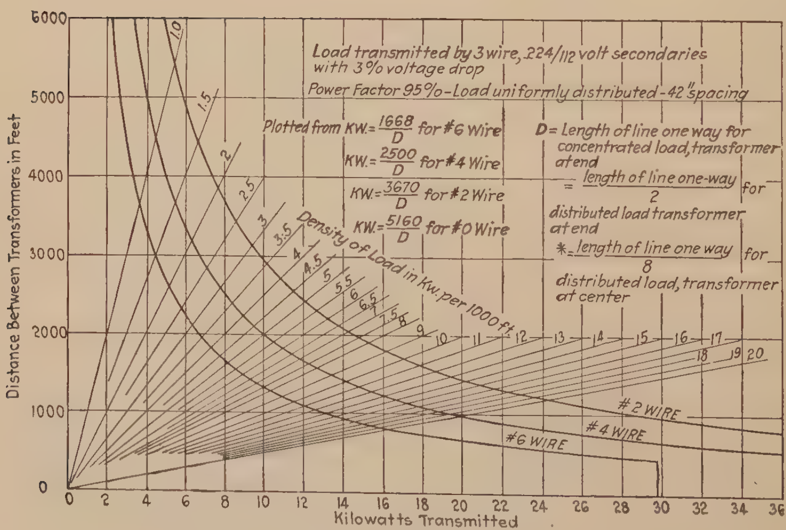

Fig. 18.—Load curves for secondaries.

of average span are taken as shown. The total voltage drop from the transformer to the end of the secondary is computed by multiplying the number of customers at each pole by the spans between that pole and the transformer and using the sum of these multiplications to find the drop in volts with the proper

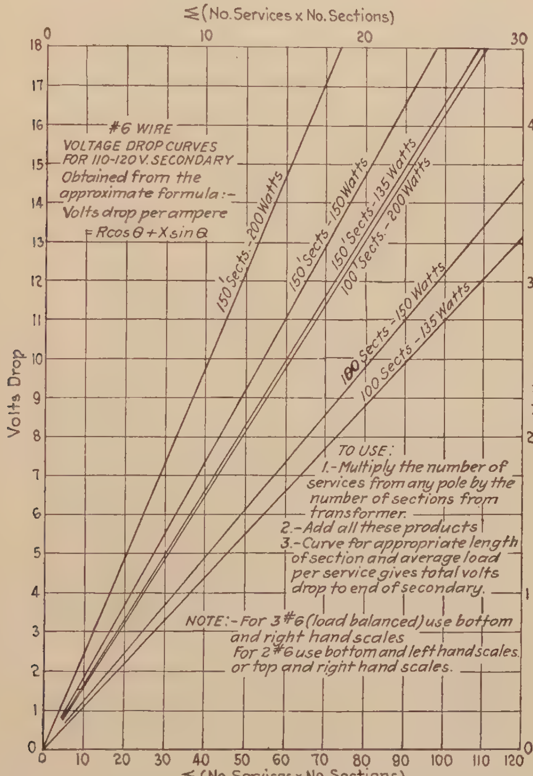

FIG. 19a.—Voltage drop curves for 110–220 volt secondaries (No. 6 wire).

curve and scale. The scales are so arranged that for three-wire secondary, *i.e.*, 220 volts with load balanced the *bottom* and *right-hand* scales should be used. For two-wire secondary, 110 volts, the *bottom* and *left-hand* scales or the *top* and *right-hand* scales give the required result.

For example consider a secondary as shown below:

Sections 150 ft., average load 135 watts

First pole	1 ×	1	=	1
Second pole	2 ×	2	=	4
Third pole				
Fourth pole	1 ×	4	=	4
Fifth pole	3 ×	5	=	15
Sixth pole	1 ×	6	=	6

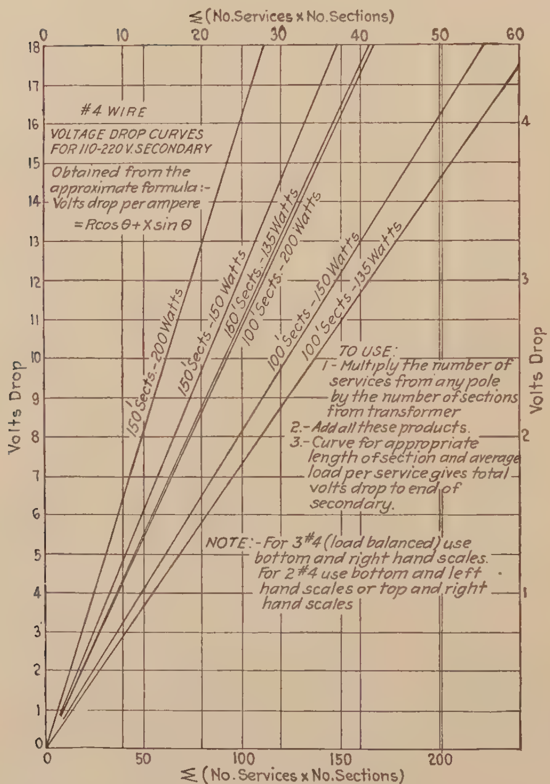

Fig. 19b.— Voltage drop curves for 110–220 volt secondaries (No. 4 wire).

Seventh pole				
Eighth pole	2 ×	8	=	16
Ninth pole	3 ×	9	=	27
Tenth pole				
Eleventh pole	2 ×	11	=	22
			Total	95

Voltage drop from curve 3.93 volts. Which is higher than it should be, $3\frac{1}{2}$ volts being about the limit to be used. In case the drop to the eighth pole is desired.

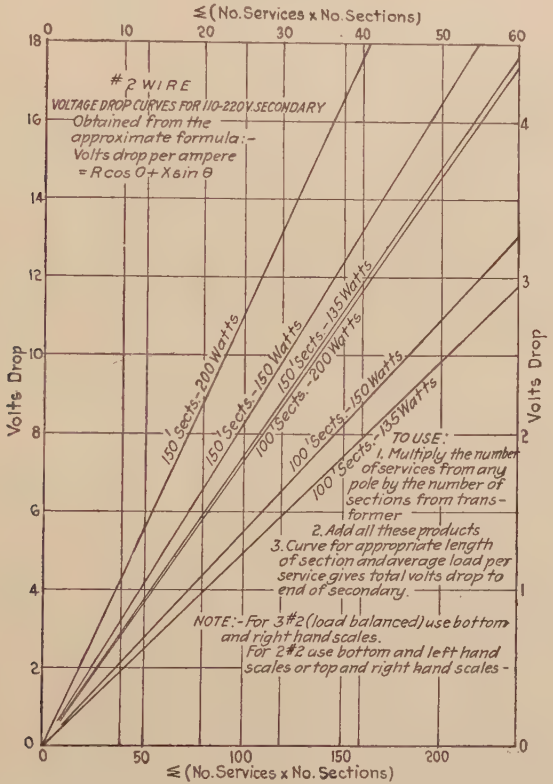

Fig. 19c.—Voltage drop curves for 110–220 volt secondaries (No. 2 wire).

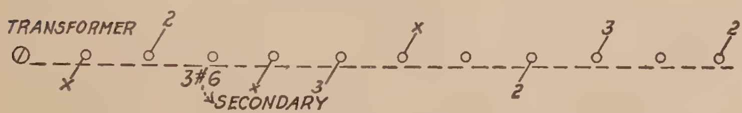

Fig. 20.

Sum of first seven above = 30
Eighth pole, 7 × 8 = 56

86

Voltage drop, 3.56 volts.

PART II

CHAPTER VIII

TRANSMISSION-LINE PROBLEMS

Method of Determining Most Economical Design for Main or "Backbone" Transmission Lines

"BACKBONE" TRANSMISSION LINES

In considering the economical design of transmission lines two general types of lines are encountered. These may be called for convenience:

1. "Backbone" transmission lines.
2. Secondary transmission lines.

To the first classification belongs that line or system of lines which forms the "backbone" of any system large or small. It is usually a line which transmits a comparatively large load (relative to the total load on the system), to some considerable distance, at a relatively high voltage. Secondary transmission lines on the other hand partake more of the nature of distribution lines. They distribute the energy from the central feeding points, served by the "backbone" line, to auxiliary stations from which it may be distributed by the ordinary distribution lines. For example, in a large system there may be several generating stations tied together by a 110,000-volt line, "the backbone." At various points along this line, substations are located, stepping the voltage down to 22,000 volts, the secondary transmission. These run to the various communities to be served where the voltage is transformed, in local substations, to ordinary distribution voltage.

Each of these types of lines present certain distinguishing characteristics which affect the method used in determining the most economical design. The problem of a backbone line is usually a specific one, *i.e.*, a definite load of known characteristics is to be transmitted from one given point to another. Future small extensions are not anticipated, these being cared for by the secondary transmission lines. On the other hand certain

conditions such as voltage, wire size, and span are not limited except by considerations of economy and good operating conditions.

Secondary transmission lines on the other hand must care for a variety of loads and distances. They are subject to short extensions to care for additional loads. They are apt to be tied together in a network. All these points must be considered and the best voltage, span, etc. to fit the *average* conditions must be adopted as a standard. Wire sizes may also be standardized to one or two sizes. Further problems are then limited to considerations of the economical load for any line, number of lines for any load, most economical routes, etc. The problems of underground transmission lines fall mostly under this classification but will be touched on separately with other underground problems.

Naturally the above division is somewhat elastic. In some systems there are no true backbone lines, the high-voltage lines being so extensive as to be similar to secondary transmission lines. On the other hand, in small systems there may be no secondary transmission lines, one or two lines comprising the whole transmission system. In such cases the backbone lines are of such a voltage, load, etc. that on a large system they would probably be classed as secondary transmission or even distribution. They are backbone lines, however, in relation to the small system, and their voltage, wire size, etc. may usually be determined by economy without any local limiting conditions. In this chapter, the problem of the backbone line will be taken up and the various elements affecting its solution will be discussed. A line of relatively high voltage and heavy load will be assumed but the same principles might be applied to any backbone line.

Having given the load to be carried, with its probable future increase, the points from and to which the load is to be transmitted, and the characteristics of that load, *i.e.*, its variations during the day and seasons, and the cost per kilowatt-hour at the generating station, the problem remains to determine the most economical route, the most economical voltage, the most economical wire size and the most economical span and arrangement of supporting structures.

It may be stated at once that no method of solving for any of these unknown quantities has yet been presented which is simple and at the same time accurate enough for the basis of a final design. Several writers have published approximate methods

of making such determinations. In these, however, the variable quantities are so numerous that it is necessary to make certain assumptions for simplicity and thus the results obtained are subject to considerable question. It is believed that these are valuable in establishing the limits of a problem but that the actual design, especially for a line of any considerable importance, should be checked up by a summation of actual cost figures as compared with the cost of several other possible alternatives.

Some of the approximate methods are given in the references below.[1]

In the paper on "Problems of 220 kv. Power Transmission" by A. E. Silver in the *A. I. E. E. Proceedings*, June, 1919, is given one of the most careful and complete studies of a high-tension transmission line, from an economic point of view, yet published. In that paper, the voltage is assumed, the economical wire size is determined from consideration of fixed charges and losses, and the most economical span determined by a complete comparison of cost figures on actual tower designs.

General Equation.—In this problem, as is the case with most economic problems, the design sought for is, in general, that one for which the total annual cost will be a minimum. The total annual cost may be expressed as follows, letting "$g \times$ (any quantity)" indicate that the annual cost is to be used.

Total annual cost = g (cost of right-of-way)
+ g (cost of towers and foundations in place)
+ g (cost of insulators in place)
+ g (cost of conductor and ground wire in place)
+ g (cost of transformers in place)
+ g (cost of lightning arresters in place)
+ g (cost of switches in place)

[1] "Economic Voltage of Long Transmission Lines," by Henry H. Plumb, *Journal A. I. E. E.*, April, 1920.

"Transmission Line Design," by F. K. Kirsten, *A. I. E. E. Proceedings*, Vol. 37, 1917, p. 685.

"Electric Power Transmission," by A. E. Still, p. 64, approximate economical voltage $= \sqrt{\text{distance} + \frac{\text{kilowatts}}{100}}$ an empirical formula.

"Notes on the Calculation of Transmission Lines for Maximum Economy," by E. Baticle, *Revue Generale de L'Electricite*, Oct. 30, 1920.

+ g (cost of special apparatus, regulators, condensers, etc. in place)
+ g (cost of substation structures)
+ annual cost of energy loss on the line
+ annual cost of energy loss on the transformers
+ annual cost of patroling, testing and other maintenance. (22)

Elements Affecting Costs.—Each of these subdivisions of cost is affected by one or more of the variable quantities whose solution is being sought, some in an extremely complicated manner. The manner in which the costs are affected will be discussed briefly.

(*a*) *Cost of Right-of-way.*—Right-of-way may have a definite cost for the whole line regardless of tower size or spacing, in which case it is affected only by the route chosen. Again, it may vary with the number of towers only, regardless of their size. In other places, it may vary both with the number and base dimensions of the towers. The locality through which the line will pass and the value of land will determine which of the above is applicable to the problem in hand.

(*b*) *Cost of Towers.*—The cost of towers is affected by a great number of conditions and its relation to the economy of a line is a difficult problem. The type of tower to be used, wide base or narrow, one, two or more circuits, etc., is of primary importance. The type of base, in turn, depends somewhat on the right of-way available. The number of circuits depends on the economical size of conductor, the limits for corona voltage on conductor and the expected increase in load. The height of the tower is also a determining factor. This depends on the minimum clearance requirements for the span, and the sag in the conductor (which in turn depends on the span, wire size and the assumptions for heaviest loading). It also is affected by the vertical spacing between conductors, which is a function of the voltage. The horizontal spacing (which varies with the voltage) likewise affects the cost, since the weight and torsional stresses are increased by longer arms. The wire size influences the cost of the tower since in most cases the tower design depends, to a considerable extent, on the maximum stress allowable on the conductor, which is proportional to its cross-sectional area. The span is also a

factor, since the longer the span, the greater the lateral wind pressure on the conductors, and this is an important factor in the design of the straight line towers. It appears to be practically impossible to develop any simple formula which would indicate the variation in tower cost with all these variable elements.

(*c*) *Cost of Insulators.*—The cost of the insulators increases with the voltage but not in direct ratio on account of the decreased efficiency per unit in long strings of insulators.

(*d*) *Cost of Conductor.*—The cost of conductor in place will be very nearly proportional to its cross-sectional area. It will, of course, depend also on the material used. It may vary somewhat with the number of towers. The cost of ground wire will in most cases be practically a constant for the line, depending only on its length, and number, size and material of ground wires decided upon.

(*e*) *Cost of Transformers.*—The transformer cost will increase as the voltage increases (but not in direct ratio) and also depends on the size of unit to be used and the type (air-cooled or water-cooled; secondary-voltage; etc.). In most problems of this kind the secondary-voltage requirements will be established and the type and size of transformer will be indicated by the load and type of substation to be used. The cost will then vary with the voltage only.

(*f*) *Cost of Lightning Arresters.*—If a given type of arrester is chosen, the cost will vary with the voltage.

(*g*) *Cost of Switches.*—If the type and size of switch is determined by the load, the cost will also vary with the voltage.

(*h*) *Cost of Special Apparatus.*—If special apparatus is needed to maintain proper voltage regulation, its cost must be included in determining economy, since it might be eliminated if large enough conductor or enough lines were used. The cost will vary with the size required, which depends upon the amount of regulation, and also with the voltage.

(*i*) *Cost of Substation Structures.*—For a given size transformer, the cost of the terminal station pertaining to the transmission line will be practically constant, varying somewhat with the voltage on account of the increased spacing of conductors. If regulators or condensers are used the cost of the substation space occupied by them must be included.

(*j*) *Cost of Energy Loss.*—The energy lost on a transmission line consists of I^2R loss due to the resistance of the conductor,

leakage, and corona loss. If the insulators are well designed and the conductor is larger than the corona limit for the voltage used, the latter two will be comparatively small and in most cases may be neglected in determining economy. The I^2R loss naturally depends on the cross-sectional area of conductor, on the material used, and on the current transmitted. The current is a function of the voltage, if the load, power factor, equivalent hours, etc. are fixed (neglecting charging current). If charging current must be considered, the average between current at source and at receiver will be sufficiently close approximation in most cases. The cost of the I^2R loss can be computed from the I^2R loss at peak load, equivalent hours, and cost per unit of energy loss. The latter two are usually fixed by local conditions and are not variables for the problem.

The energy losses on transformers consist of core losses and copper losses. Since transformer efficiencies do not vary greatly with size or voltage, the variations in the losses will not be considerable. In general these losses decrease somewhat as the size of transformer increases and increase slightly as the voltage increases. The exact amount of variation must be determined for the particular transformers to be used. Other energy losses will be encountered if special apparatus such as regulators or condensers are used and the cost of this loss must be included.

(*k*) *Cost of Patroling, etc.*—The cost of testing insulators will depend somewhat on the number of units used in a string and hence on the voltage. The cost of patroling will vary with the length of the line and the nature of the country to be covered. Other maintenance and repair costs can only be estimated and will probably be roughly proportional to the length of the line.

Solution by Equation Impracticable.—The above is an indication of the various number of elements which affect the cost of each of the factors that go to make up the total annual cost on a transmission line. It is evident, that, with so many variable quantities, there is no simple solution for minimum annual cost, which will be general for all cases. It is true that, for any given problem, when all the conditions are fixed excepting the four chief variables mentioned above, *i.e.*, voltage, wire size, tower spacing, and route, equations can be written for each of the elements of cost in terms of these variables, which will approximate quite closely that cost under any condition. For example, the

cost of a given type of transformer can be represented with sufficient accuracy by the expression

$$\text{Cost} = K_1 + K_2\, E^n$$

where K_1, K_2 and n are constants and E is the voltage. Some of the costs are very difficult to represent, the cost of towers for example. If all these expressions are determined, however, and are combined into the general equation of annual cost, it will be found that this equation is so complicated and contains the variables in so many different powers, that a solution is impossible except by trial. There is then no apparent advantage over the method of assuming a number of alternative designs, using definite values for each of the variables, and computing the actual annual cost of each of these designs. A comparison of these costs will indicate the most economical.

Method of Solution Outlined.—In order to systematize the computation and to facilitate the determination of the effect on the total cost, of changes or additions to any design, the following method is suggested.

1. *Fixed Quantities.*—As many as possible of the elements affecting the problem should be fixed. These will be:

(*a*) The maximum load. The expected increase should also be determined. In case of more than one feeding point each load must be considered separately.

(*b*) The load factor and equivalent hours of the load.

(*c*) The power factor of load.

(*d*) The cost of energy per kilowatt hour.

(*e*) The length and cost of right-of-way (at least approximately) for the various possible routes.

(*f*) The type of tower to be used for each route.

(*g*) The per cent annual charges applicable to the various kinds of property entering into a transmission line.

2. *Limits of Problem.*—It is essential to discover approximately the limits of the problem so that time will not be wasted in considering voltages and wire sizes which are far from the final result. In case the designer's experience is not sufficient to tell him this, the approximate formulas found in the references given above will be found useful.

3. *Cost Data.*—The necessary cost data, quotations, etc., must be collected. These will be:

(*a*) Cost of transformers, of the size determined by the load for various voltages within the range of the problem (more than one size may be neces-

sary especially if there are several feeding points with different sized loads.) Core loss and copper loss should also be ascertained.

(*b*) Cost of switches, of proper size, for various voltages.

(*c*) Cost of lightning arresters, for various voltages.

(*d*) Cost of insulators, for various voltages. The limit in mechanical load for any string of insulators should also be determined. In cases where pin insulators are considered in comparison to suspension type, the cost of both must be obtained. The cost of placing insulators should be included.

(*e*) The cost per pound of conductors of different materials—copper, copper-clad steel, and aluminum steel. In case the price per pound varies considerably with the size, the cost for a variety of sizes within the range of the problem should be obtained. The cost of stringing the conductor should be estimated or determined from previous experience.

(*f*) The cost of various sizes of tower for each type. If possible sufficient costs on towers should be obtained that the variation of cost with span, wire size and voltage may be determined. This could be accomplished if three voltages, three wire sizes, and three different spans, covering the probable range of the problem, are considered. By plotting curves, the intermediate values could be determined with sufficient accuracy. The cost of foundation should be included with each tower. Anchor towers and semi-anchor towers should also be considered and similar costs obtained.

(*g*) The cost of terminal substations of the required size. The variation in this cost with voltage should be determined, if possible. In case other special terminal apparatus, such as condensers or regulators are considered, the additional substation cost for these must be estimated.

(*h*) Quotations on special apparatus considered, regulators, condensers, etc., should be obtained with variations in this cost with size and voltage.

(*i*) Annual cost of maintenance, testing, patroling, etc. must be estimated.

4. *Arrangement of Cost Data.*—After the data is collected, it should be arranged for convenient use. This probably can best be done by means of curves. Such curves are illustrated in the accompanying figures. They should give:

(*a*) Annual cost of transformers (in place) in terms of voltage (Fig. 21).

(*b*) Annual cost of switches (in place) in terms of voltage (Fig. 22).

(*c*) Annual cost of lightning arresters (in place) in terms of voltage (Fig. 22).

(*d*) Annual cost of insulators per string in terms of voltage.

(*e*) Annual cost per mile of conductors of different materials in place in terms of cross-sectional area.

(*f*) Annual cost of towers in place. These can be arranged (for example) as a series of curves for each standard voltage showing for each standard wire size (and material) the variation of cost with span.

(*g*) Annual cost on terminal substation in terms of voltage.

(*h*) Annual cost of transformer energy losses in terms of voltage.

(*i*) Annual cost of line energy losses per mile for each material of conductor considered, in terms of cross-sectional area.

5. *Most Economical Design.*—It now remains to apply these costs to determine the most economical design. The route is usually the most limited of all the variables, there ordinarily being no more than two or three possible routes at most. Like-

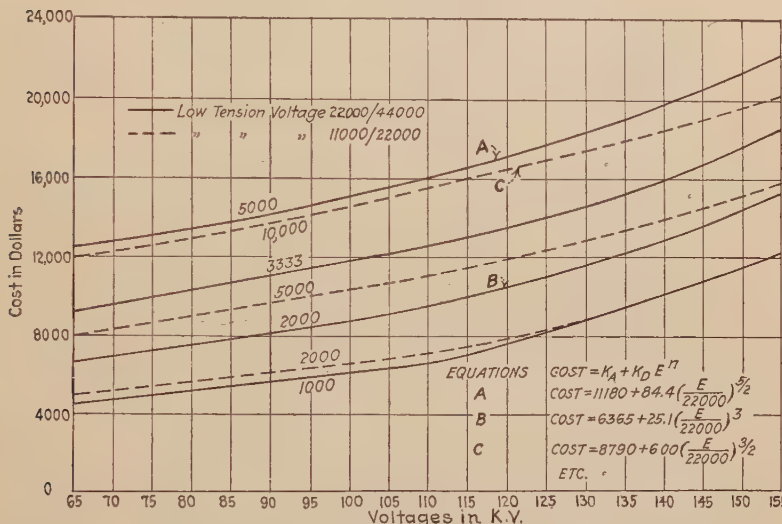

FIG. 21.—Cost of single-phase 60-cycle transformers.

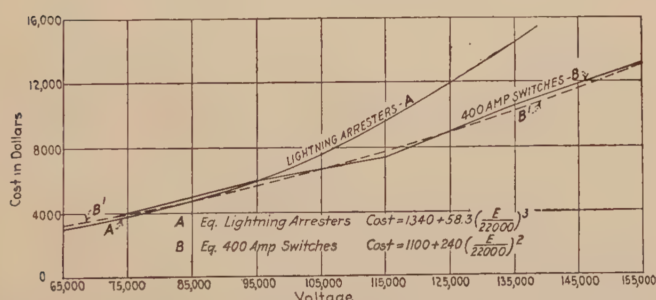

FIG. 22.—Cost of lightning arresters and switches showing variation of cost with voltage.

wise, on any route, the layout is somewhat limited. Locations of anchor towers and semi-anchor towers are usually more or less fixed. On straight runs the span can often be varied. If two or three possible layouts are made for each route, varying the span where possible, a sufficient comparison will be obtained.

If, now, for each of these layouts, a sheet of curves is prepared, showing for each standard conductor considered, a curve, giving the variation in total annual cost with the voltage, a complete and accurate study of the economy may be made. Not only will the most economical voltage, wire size, route and layout be obtained, but the comparative effect of variation in these quantities may be studied. Curves may be added to show comparison of two or more circuits with a single circuit. If a design, chosen tentatively, proves, on further investigation, to be impracticable on account of corona limit, too great regulation, etc., the cost of correcting the fault by a change in design, addition of regulator, etc., as compared with one of the other designs not having that fault, may be easily determined.

Space does not permit dwelling on the subject of "backbone" transmission lines more in detail. Any problem of this kind is one which will bear an almost infinite amount of study. Naturally the detail to which such a study should be carried will depend somewhat on the size and nature of the project. As a rule, however, time spent on an economical determination is well repaid. In many cases some of the variables are limited—voltage may be determined by other than economical considerations, route and location for towers may be fixed, etc. This simplifies the problem but in any case some such study as that outlined above is essential to an accurate determination. Variations in the method will, of course, be found to accommodate special conditions. In any case, it is to be emphasized that the problem is a complicated one and usually does not bear solution by any easy, approximate method.

CHAPTER IX

TRANSMISSION-LINE PROBLEMS

SECONDARY TRANSMISSION LINES

Determination of Most Economical Standards of Construction, Conductor Size, Loading, Route, Etc. on Lesser or Secondary Transmission Lines

In the preceding chapter there were discussed the points of distinction between "backbone" transmission lines and "secondary" transmission lines. It is purposed here to consider the problems arising in connection with "secondary" transmission lines and to indicate their economic solution.

The "secondary" transmission lines of any system must be considered as a class instead of as one or more specific problems. Their service is varied. They are called upon to feed loads of various sizes, and various power factors and load factors. They may have a considerable diversity in lengths, types of route, etc. It is obviously impracticable to consider each line as a special problem. It is necessary to adopt certain standards which will best fit all cases on an average, allowing some variation if necessary in wire sizes, etc. There then remains the problem of how best to adapt these standards to fit any given condition.

In this study, therefore, there are two separate divisions. The first is of a somewhat similar nature to that of the "backbone" transmission line, excepting that the problem is general rather than specific. It deals with the establishment of standards which will best fit average conditions, such as a standard voltage, one or more standard wire sizes (the variety depending somewhat on the range of loads handled) and a standard span. The second division includes problems relating to the proper use and combination of these standards. Such problems as the most economical route, the economical division of load among several lines, the load at which wire size should be changed or an additional line run, and reconstruction problems are in this class.

It would be impossible in a reasonable amount of space to attempt to cover in any detail all the problems which might arise in connection with secondary transmission lines. The field of study is very large and new problems are constantly appearing. Some of the problems commonly met with under each of the two

divisions will be discussed, however, and the elements affecting their solution will be indicated.

The general method of attacking any problem of this kind is to determine an expression for annual cost and then to discover under what conditions that cost will be a minimum. Since the results are to be of general application, it is necessary that the data used be general, for the whole system. Average costs of pole line per mile should be used, for example, rather than an estimated cost for a certain line over a given route. Similarly, the results obtained should be displayed in formulas, curves or tables which may be of general application.

Data Necessary.—The data necessary for solving such problems on secondary transmission lines include the following:

(*a*) Range of sizes of loads considered.

(*b*) Characteristics of various types of load carried, load factor, power factor, equivalent hours, etc. If the characteristics of the general types of load are thoroughly understood any individual load can be studied as a combination of several general types in different amounts. A load may consist of part lighting, part power, part street railway, etc. and its individual characteristics are a combination of the characteristics of all these types.

(*c*) Average material and labor costs on standard construction; average right-of-way costs if such figures are obtainable.

(*d*) Cost of energy for various types of load in various parts of the system.

Standard Voltage.—The solution of the first class of problems mentioned above is very often limited by other considerations than strict economy. Extensive systems rarely spring into being suddenly. Usually they are developments from comparatively small beginnings. Hence, what was yesterday the backbone transmission line of the small system, becomes today part of the secondary transmission network of the large system. It happens, therefore, that the secondary transmission voltage is rarely chosen as such, but is rather a development from past practice. There is always the question of the economy of changing the standard voltage, usually to one higher. This can only be determined by a careful study of present and probable future conditions, taking into consideration the additional cost for transformers and station equipment and the cost of making the change-over, as compared with the saving in line losses, the smaller conductor used, and the increase in capacity in the system with the proposed higher voltage. Such a change can sometimes be made economically, where single-phase transformers are used to a large ex-

tent, by changing from a delta to a star connected system. In cases where the voltage can be chosen in advance on the basis of economy, an analysis similar to that suggested for "backbone" transmission lines will be necessary.

Standard Span.—The standard span to be chosen will depend largely on the type of construction used. Conditions may vary from the use of a steel-tower line on private right-of-way, to that of wood-pole lines along the highway, carrying distribution in addition. For the first, probably the most advantageous span can be chosen somewhat as outlined for "backbone" lines and need not necessarily be uniform nor the same for all lines. For the latter, the span will be determined partly by the strength of the poles and partly by the needs of the distribution lines. These spans can be fairly uniform, not exceeding a definite maximum of 150 to 250 ft. ordinarily.

Conductor Size.—The question of standardization of conductor size will depend largely on the kind of loads carried and their distribution over the system. In some cases it will be found advantageous to standardize on one or two sizes and care for additional load by more lines. In others, it may be better to use a wider variety of conductors, accommodating the size to the load carried. A method of studying conductor economy is given below. By its means, the most economical wire size for any load may be chosen and, under proper conditions, standard sizes may be fixed. This method of study also aids in the solution of several of the problems of the second class as will be pointed out later.

Example of Study of Conductor Economy.—Let a standardized type of construction with given voltage and standard span be assumed. The total annual cost of such a transmission line is composed of:

1. Annual cost on construction exclusive of conductor.
2. Annual cost of conductor in place.
3. Annual cost of energy loss.

For this example, a wood-pole line will be assumed with a maximum span of 175 ft. to accommodate distribution.

The annual cost of poles and fixtures per 1,000 ft. of line will, of course, vary with the wire size, if the line is properly designed for safety, probably after some such formulas as $K_1 + K_2A$, where K_1 and K_2 are constants and A is the cross-sectional area of the conductor.

The annual cost of the conductor per 1,000 ft. will vary approximately with A as also will the cost of stringing, *i.e.*, cost in place = $K_3 + K_4A$.

The annual cost of energy loss per 1,000 ft. will vary with the square of the load, the first power of the equivalent hours, and the cost of energy, and inversely with the cross-sectional area.

Then the total annual cost per 1,000 ft. of line

$$Y = K_1 + K_3 + (K_2 + K_4)A + K_5 \frac{kw^2 t C_e}{A} \tag{23}$$

Where t = equivalent hours, C_e cost of energy loss per kilowatt-hour, kw = load in kilowatts

Simplified

$$Y = K_a + K_b A + K_c \frac{kw^2 t C_e}{A} \tag{24}$$

Where $K_a = K_1 + K_3$ and $K_b = K_2 + K_4$, and $K_c = K_5$

Most Economical Conductor Size.—The most economical conductor size will be that for which the value of Y is a minimum. This can be obtained by setting the first derivative of Y with respect to A equal to 0

$$\frac{dY}{dA} = K_b - K_c \frac{kw^2 t C_e}{A^2}$$

$$A = kw \sqrt{\frac{K_c}{K_b} t \; C_e} \tag{25}$$

Most Economical Load.—If it is desired to determine the most economical load for any line, the derivative of Y must be taken with respect to the load, kw. It is evident that, with the equation in the above form, the result would be $kw = 0$, since the minimum line loss is obtained with no load. However the line must be considered as a working unit. Hence, if the above expression is changed to represent the annual cost per kilowatt transmitted, and the minimum value of that quantity found, the most economical load for the line will be discovered.

If Y' = the annual cost per kilowatt transmitted

$$Y' = \frac{K_a + K_b A}{kw} + \frac{K_c kw t C_e}{A} \tag{26}$$

$$\frac{dY'}{dkw} = -\left(\frac{K_a + K_b A}{kw^2}\right) + \frac{K_c t C_e}{A}$$

$$kw = \sqrt{K_a + K_b A \left(\frac{A}{K_c t C_e}\right)} \tag{27}$$

Numerical Example.—The values of the constants must be obtained to accord with local conditions. K_c is evaluated as follows:

Y_e = annual cost of energy loss per 1,000 ft. =

$$3\,I^2R \times t \times 365 \times \frac{C_e}{1{,}000}$$

Where I = current per wire

R = resistance per 1,000 ft., one wire,

$$R = \frac{\rho \times 1{,}000}{A} \quad \text{Where } \rho = \text{resistivity per mil foot}$$

$$I = \frac{kw \times 1{,}000}{\sqrt{3}\,E \cos\theta}$$

$$\text{Then } Y_e = \left(\frac{1{,}000kw}{E\cos\theta}\right)^2 \times \frac{1{,}000\rho}{A} \times t \times 365\frac{C_e}{1{,}000} = \frac{K_c kw^2 tC_e}{A}$$

$$\text{Where } K_c = \left(\frac{1{,}000}{E\cos\theta}\right)^2 \times \rho \times 365$$

$$\text{If } \rho = 10.8 \qquad K_c = \frac{3{,}940 \times 10^6}{(E\cos\theta)^2}$$

Let us assume for example a 22,000-volt line.

For which $K_1 = 39$

$$K_2 = \frac{12.2}{10^5}$$

$$K_3 = 1.715$$

$$K_4 = \left(\frac{2.21 + 147.5C_{cu}}{10^5}\right) \qquad \text{Where } C_{cu} = \text{cost of conductor per pound}$$

$$= \frac{31.7}{10_5} \qquad \text{If } C_{cu} = 0.20$$

$$K_5 = 11.3$$

$$K_a = 40.72$$

$$K_b = \frac{43.9}{10_5}$$

$$K_c = 11.3$$

(From Eq. 24) $$Y = 40.72 + \frac{43.9}{10^5}A + 11.3\frac{kw^2tC_e}{A}$$

(From Eq. 25) $$A = kw\sqrt{\frac{11.3 \times 10^5 tC_e}{43.9}} = 160.3kw\sqrt{tC_e}$$

(From Eq. 26) $$Y' = \frac{40.72 + \frac{43.9}{10^5}A}{kw} + 11.3\frac{kwtC_e}{A}$$

(From Eq. 27) $$kw = \sqrt{40.72 + \frac{43.9}{10^5}A\left(\frac{A}{11.3tC_e}\right)}$$

The foregoing are all simple equations in terms of the load carried, the cross-sectional area of conductor, the equivalent hours and the cost of energy. The latter two quantities are more or less inter-related. A study of energy cost will show that the cost per kilowatt-hour varies, among other things, with the load factor and hence with the equivalent hours. The value of tC_e may then be obtained approximately for any value of t with any type of load.

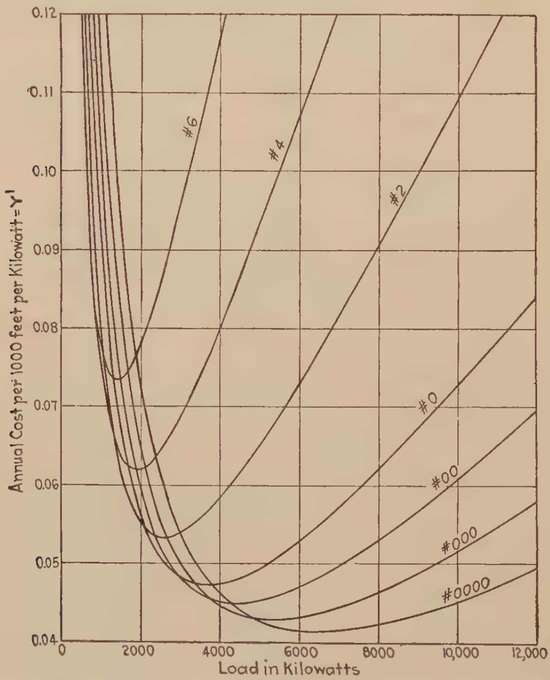

FIG. 23.—Annual cost per kilowatt per 1,000 ft. of 22,000-volt transmission line (single circuit—tC_e = .06 j cos θ = .85).

It will be found advantageous for study to plot curves of all the equations given above. If different values of tC_e are chosen, as $tC_e = .02$, $tC_e = .04$, $tC_e = .06$, $tC_e = .08$ and $tC_e = .10$ covering the desired range of values and a sheet made up for each, giving a curve, plotted between annual cost, Y, and load kw, for each wire size, the first equation (24) is well displayed; similarly Eq. 26. For Eqs. 25 and 27, a curve, plotted between A and kw for each desired value of tC_e, may be obtained.

For a numerical example the value of equivalent hours is taken

as 6 and the corresponding value of tC_e assumed as .06. The curves for Y', the annual cost per kilowatt per 1,000 ft., are plotted as shown in Fig. 23. The curves for Y, the total annual cost per 1,000 ft. are easily obtained if desired but are not shown

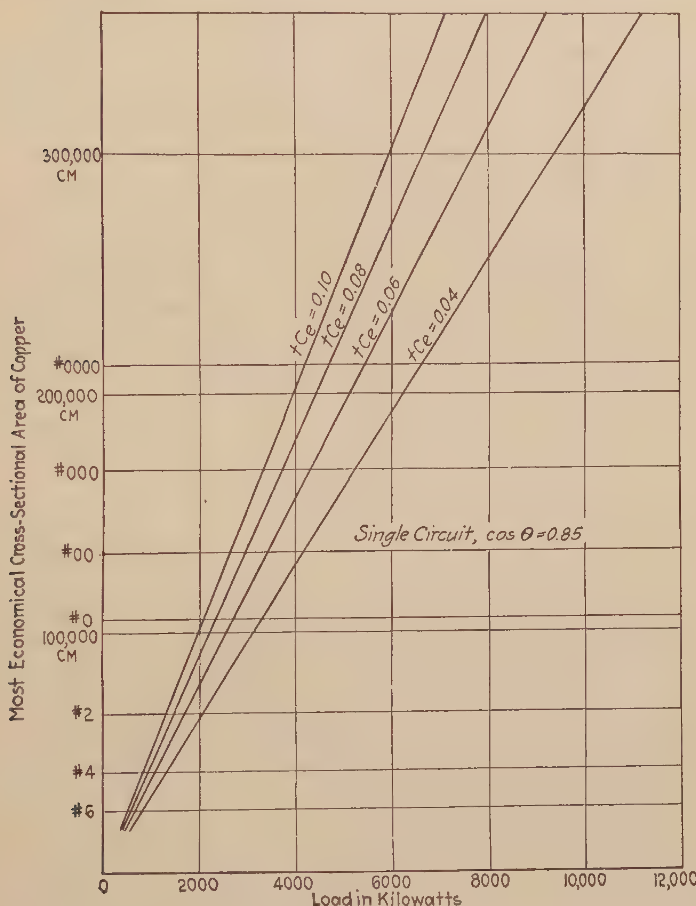

FIG. 24.—Most economical conductor size for 22,000-volt transmission line.

here. These curves for Y' are interesting in that they show definitely the point of minimum cost or the load of greatest economy for any given conductor size. Likewise, they show, for any load, the most economical conductor and just how much cheaper one size is than another for any load.

Figures 24 and 25 are also plotted from Eqs. 25 and 27, using

various values for tC_e. Figure 24 gives the most economical conductor size for any load and Fig. 25 the most economical load for any wire size. The curves for $tC_e = .06$ might have been derived directly from Fig. 23, Fig. 24 being a bounding curve for all the curves shown and Fig. 25 the locus of all the minimum points. It is interesting to note that the most economical size of wire for any load is not the same as the size for which that is the most economical load. The reason for this is easily seen from Fig. 23, for while at 2,500 kw. No. 0 wire appears to

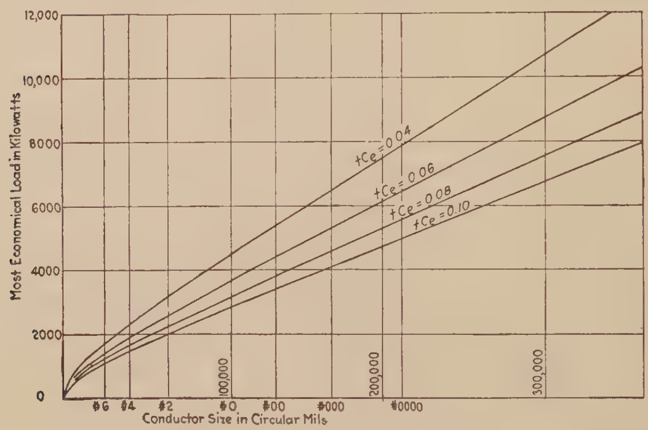

Fig. 25.—Most economical loading for 22,000-volt transmission line.

be most economical, with an annual cost of .051 per kilowatt per 1,000 ft., if a No. 0 line is loaded to 3,800 kw., the annual cost will be only .0473 per kilowatt per 1,000 ft.

Use of Curves.—By means of such curves as these, a number of the problems of secondary transmission lines may be readily solved. If the size and characteristics of a load are known, the most economical conductor may be found from the proper curve. If some other size than the most economical is used, its additional cost is given by the curves. If standard sizes are to be determined, the range of loads, and their characteristics, to be handled may be studied in connection with the curves and the most economical size or sizes to fit the majority of cases may be chosen. If the problem is one of changing conductor sizes, the load at which the annual cost after the change (including the annual charges on the cost of changes) will be less than the annual cost with the present size, may be taken from such

curves as Fig. 23. Similarly, if curves for two (or more) circuit lines are prepared, the economy of adding a second circuit, of building an additional line, etc., may be studied. (This problem usually involves considerations of reconstruction, see Chap. X.) If several lines of different sizes are feeding one station, the economical division of load is shown by curves similar to Fig. 25.

Most Economical Route.—Another problem which is encountered in the design of nearly all transmission lines is the choice of the best route. It is seldom possible to follow anything like a direct "air line" route on account of the difficulty in obtaining right-of-way. If wooden poles or steel structures with small base are used, a route is ordinarily chosen which follows the highway in such a way as to arrive at the destination with the least possible length of line and the fewest possible difficulties in construction. It often occurs, however, that this choice is not a simple matter. There may be two or more routes, each of which has advantages and disadvantages which more or less offset each other, and there is no self evident choice between them. It is then necessary to make a careful cost analysis to determine the most economical route. The relative economy of the various routes can be weighed against any other features which are not subject to a tangible cost comparison, and the most practicable route will usually be made evident.

Probably one of the most usual and simple problems of this kind is the choice between two routes, one of which is shorter than the other but necessitates the purchase of a certain amount of right-of-way, while the other is longer but follows the highway and is not subject to right-of-way charges. It is the purpose here to show how the cost of two such routes can be readily compared and a general equation derived which will cover all such problems on any particular system. This equation can be used to determine curves that give a more tangible method of choosing the best solution.

As a corollary to this problem we have the condition where there is a choice of a longer route, clear of trees and other obstructions, and a shorter one which will require extra high structures, additional guying, etc. In fact the problem resolves itself into the question of "How much can we afford to spend in addition to the normal cost of construction in order to shorten a route?"

The basis for such a cost comparison will naturally be the annual cost of the two routes. If we can determine what will be

the saving in annual cost of the shorter route over the longer when only normal line construction cost is considered, we can then tell what is the maximum amount which we would be justified in expending for right-of-way or additional material, etc., in order to use that shorter route. If the longer route should also require some extra construction expenses these should naturally be included in the comparison.

The annual cost of any route is made up of the annual charges against the construction itself and the annual charges for the energy loss, the latter depending on the load carried and the size of wire used. It is first necessary to determine as accurately as possible what the unit annual charges will be on a normal line, *i.e.* one without exceptional difficulties.

Determination of Normal Line Cost.—The annual charges against the construction will depend upon the standard type of construction used and on the cost of materials and labor in the locality under consideration. There must be obtained the total cost per mile of the line in place including both material and labor costs on poles, crossarms, pins, insulators, wire, grounds, guying, etc. In doing this, the sizes and quantities of the various materials which would constitute an average normal mile of line must first be determined. The annual cost may then be obtained by figuring interest, taxes and depreciation on this first cost allowing for the difference in depreciation between the different materials used. A certain amount per year should also be added for maintenance, patrolling, etc. The resulting annual cost will be a constant for any condition of loading, providing the size of wire is fixed, and may be represented by K_6.

Usually when a shorter route is selected a number of corners are eliminated. The cost of a corner construction, especially on high-tension lines, may be quite an appreciable addition to the normal cost of the line. It can be figured however in the same manner as the normal line cost. This annual cost per corner may be represented by K_8.

Annual Cost of Energy Losses.—The annual charges due to loss of energy may be determined as shown above in the study of conductor economy.

Then,

Annual cost of energy loss $= K_7 kw^{2\prime\prime}$ per mile.

$$\text{Where } K_7 = \frac{365{,}000\, r\, t\, C_e}{(E \cos\theta)^2}$$

Where r = resistance of conductor in ohms per mile.

When these constants have been evaluated for the particular conditions of the problem in hand, it is then possible to determine the total annual cost per mile with normal line construction and the total annual cost for any particular length including the corners.

Determination of General Equation.—If we let L represent the difference in length between the shorter route and the longer, the savings in annual cost of the shorter route over the longer would be L times the annual cost per mile if normal line construction only is considered. This, then, represents the maximum amount which it would be economical to spend in addition to the normal cost of the line in order to utilize the shorter route. If the additional cost is for right-of-way, this annual amount is the maximum rent which we could afford to pay for it or if the right-of-way is bought outright, this amount represents the interest and taxes on the maximum amount which could be paid for the necessary land. In the case where the additional cost would be for extra poles and other line material, the above savings would represent the interest, taxes, and depreciation on the maximum amounts which could be so expended.

If L = the length in miles which would be saved by using the shorter route,

C_1 = the maximum extra expenditure allowable in order to use the shorter route,

C = the extra expenditure per mile saved $= \frac{C_1}{L}$,

N = the number of corners saved by using the shorter route,

g = per cent annual charges on additional expenditure (interest, taxes, depreciation),

K_6 = the annual cost of normal line construction per mile,

K_7kw^2 = annual cost of energy loss per mile,

K_8 = annual cost of a corner in addition to normal line cost,

Then

The annual charges on the additional expenditure for right-of-way, extra poles, etc. would be gC_1

$$gC_1 = L(K_6 + K_7kw^2) + K_8N \tag{28}$$

which is a general equation and can be applied to any such case if the proper values for the various constants are obtained. If then the contemplated shorter route can be used with a less expenditure than C_1 it would be an economical proposition to use it.

The above equation can be made even more general and be shown graphically by means of a curve if we reduce it to terms of C or the maximum allowable expenditure per mile of length saved, omitting for the present the amount saved on corners.

$$C = \frac{K_6 + K_7 kw^2}{g/100} \tag{29}$$

The value of g as indicated above depends upon the nature of the additional expenditure. If it is for right-of-way, interest and taxes must be charged but no depreciation need be considered. We may take interest at 6 per cent., taxes at 2 per cent or g = 8 per cent. This represents yearly rental. If extra line material is to be purchased depreciation also must be added. Considering the life of such material as 20 years, depreciation = 5 per cent and g = 13 per cent.

Data for Curves.—The accompanying curves, Fig. 26, were plotted assuming values for K_6, K_7 and K_8 as follows:

$$\begin{aligned}
K_6 &= \$350 \text{ (wood-pole construction)} \\
r &= .539 \text{ for No. 0 wire} \\
t &= 6 \text{ hr.} \\
E &= 46{,}000 \text{ volts} \\
\cos\theta &= .90 \\
C_e &= .01 \text{ per kilowatt-hour} \\
K_7 &= \frac{365{,}000\, rtCe}{(E\cos\theta)^2} = \frac{0.69}{10^5} \\
K_8 &= \$40 \\
g &= 8 \text{ and } 13
\end{aligned}$$

$$C = \frac{320 + \dfrac{.69\, kw^2}{10^5}}{.08} \quad \text{and} \quad C = \frac{320 + \dfrac{.69\, kw^2}{10^5}}{.13}$$

These equations are plotted for various values of the load, kw. The lower curve A is plotted for g = 8 per cent or when the additional expenditures to be made in using the shorter route will be for such property as right-of-way which has no depreciation.

In case the right-of-way is obtained on a yearly rental basis, 8 per cent of the amount shown by the curve would be the allowable yearly rent. The upper curve is to be used when the extra cost will be for extra line material such as poles, stubs, guys, etc. for which the life was assumed to be 20 years and $g = 13$ per cent.

If any corners will be saved by using the shorter route, the cost of these may be added to the amount shown by the curve or

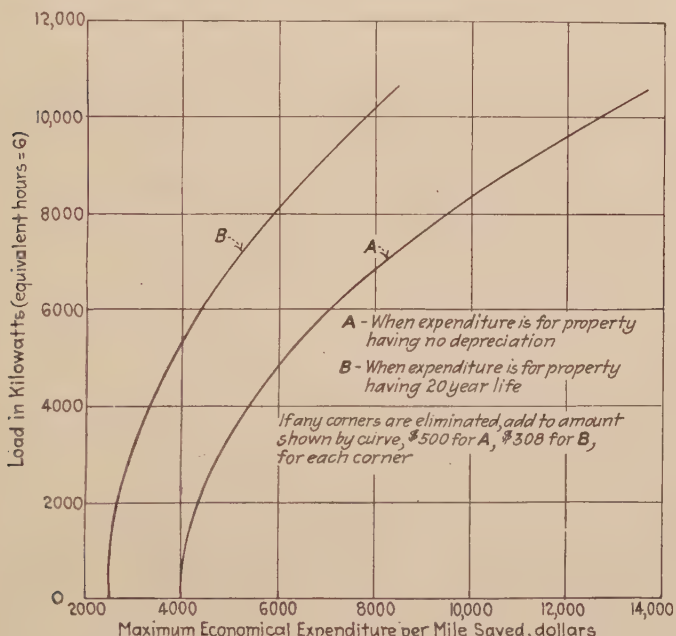

FIG. 26.—Curves showing maximum economical expenditure in addition to normal line cost in order to shorten a route.

$\frac{K_8}{g/100}$ for each corner. If K_8 = \$40, this amounts to \$500 for g = 8 per cent and \$308 for g = 13 per cent.

Discussion of Curves.—These curves, then, show the amount which could be spent in constructing a line in addition to the normal cost of construction under average conditions in order to shorten the length of the route 1 mile. The value of C when the load is O represents annual charges on the normal construction cost of 1 mile of line capitalized at the percentage g, which is the percentage of annual charge against the additional property

acquired and depends on its nature. As the load increases the value of C increases due to the increasing cost of the line loss.

The slope of the line away from the vertical illustrates graphically the importance of knowing as accurately as possible the load and the load factor that it is intended to carry on the line, in order to correctly make the choice of routes. Though an accurate determination may be practically impossible, it is usually possible to work between some assumed limits that will represent a wide range of conditions and still lend themselves to analysis by means of the curves.

Furthermore, as the load increases, the point which stands out plainly is the great economy which will ordinarily be achieved by the use of a shorter route. For example if right-of-way can be obtained for a yearly rental of \$1 per pole, at 8 per cent this would represent \$12.50 per pole or at 35 poles per mile, \$437.50 per mile. With a load of 4,000 kw. the curves show that we could spend, according to curve A, \$5,300 for each mile saved, which would purchase right-of-way, at the above rate for 12 miles. Or, from curve B we could expend \$3,300 for additional material in order to save 1 mile. If any corners were eliminated the amount would be still more. Hence, if the actual expenditure necessary is any less than the above figures, the difference would represent a real saving and it would be economy to use the shorter route.

Naturally the curves here given could not be used for any but the particular type of construction and voltage for which they were computed. It is a simple matter, however, to develop similar curves to fit any other case from the equations given. The economy shown by such a curve, however, cannot be used as an absolute criterion in the choice of a route. There are other factors which do not lend themselves to such an exact cost analysis. The matter of patrolling a line and making repairs might be considerably more expensive along private right-of-way than along a main highway. The proximity to a railroad might considerably affect the cost of erection. The matter of protection against severe storms must be considered. Many other details will, in any construction project, present themselves for analysis. If, however, we have a concrete comparison of the relative economy of one route over another, we have gone a long way toward an exact determination of which will be the most advantageous under all conditions.

As brought out at the beginning of this discussion, only one specific point in economies of routes has been covered here, that is, where the choice between two routes is to be made, the shorter one requiring purchase of right-of-way or installation of higher structures and special reinforcements. The many other problems in the choosing of the economical route for a transmission line could be treated in a very similar manner. The question of the economy of diverging from the shortest route in order to utilize old poles already in place is taken up in the next chapter under "Reconstruction Problems."

The above will give an idea of the kind of problems in connection with a secondary transmission system which require a solution for economy. It must always be kept in mind, of course, that economy is not the only consideration in designing a line. Mechanical strength is an important feature. Good regulation is essential. While the economical conductor size is usually independent of the length of the line, regulation depends on the length. Often, on a long line, a larger size than the most economical must be used to give good voltage. In case artificial means of improving regulation are considered, their cost will tend to offset the economy of the smaller conductor and a study of the line as a whole including all such items is necessary. The problem is still one of economy. In any case the determination of the most economical conductor size, voltage, span, route, etc. will serve as a starting point for the study of the most advantageous design for final adoption.

CHAPTER X

RECONSTRUCTION PROBLEMS

Principles Involved in the Solution of Problems Dealing With the Alteration or Reconstruction of Lines Already Built —Method of Including Value of Salvaged Material in Cost Study

A great number of the problems in transmission and distribution lines, involve the consideration of reconstruction, that is the improvement or enlargement of a system already in operation. For example, it may be desired to determine the advisability of shortening a line already in operation by rebuilding part of it over a new route. Or, it may be necessary to meet an unforseen increase in the load carried, by rebuilding an old line, increasing the size of the wire, adding an additional line on the same poles, or changing the voltage. While this chapter deals primarily with transmission line reconstruction, the principles enumerated may be applied to such problems on any part of the system.

For convenience, the term "reconstructed line" will be used to designate the line after the change is made. In any such case, it is necessary to consider the fact that the old line has more or less of its serviceable life left and the value of this must be added to the cost of the new construction in determining the total investment represented by the finished line. This total is the amount upon which the reconstructed line must pay a return and should be justified by the increased capacity secured. Or, in a problem of economy, such as shortening a line, it is the amount upon which the annual cost of the new installation must be figured. This annual cost must be less than that of the old line if there is to be any advantage in the change.

Total Investment Represented in Reconstructed Line.—In order to determine the total annual cost of the reconstructed line or section of line, it is easier to first consider the factors which affect the total investment involved, either to increase or decrease it. It will at first be assumed that the new line is to be entirely of new construction and the old line is to be salvaged. Later, the use of old material in the new line will be discussed. It is evident that, for the purpose of this analysis, the amount of investment represented by the old line as it stands must be computed on the principle of the cost to reproduce it in its

present condition, at present prices. It is to be replaced by new construction at present prices and the comparison of annual costs of old and new must be on the same basis. This amount may be obtained by figuring the detailed cost of such a line, including material and labor, at present prices and subtracting from this a percentage of the whole equal to the fraction of the total assumed life of the line which has already elapsed. The value of the elapsed life is assumed to have been paid for in the return from the operation of the line up to the present time. If, therefore, a new line is built to replace the old and the old line is removed, there is added to the total investment the cost of all the new material used and the labor involved in the change. There is subtracted from the total investment the salvage value of the material recovered. The investment represented by the old line as it stands is thus increased by:

1. The cost of the material used in the reconstructed line.
2. The cost of the labor necessary in the building of the reconstructed line.
3. The cost of the labor necessary in dismantling and removing to the warehouse the material in the old line.

The investment is decreased by:

The present value of the material in the old line, (*i.e.*, its cost if new, less a percentage of depreciation for age).

Hence, the

$$\begin{aligned}\text{Net increase in investment} = {} & \text{Cost of material in reconstructed line} \\ & + \text{Cost of labor in reconstructed line} \\ & + \text{Cost of labor removing old line} \\ & - \text{Present value of material in old line.}\end{aligned}$$

As was explained above,

$$\begin{aligned}\text{Investment represented by the old line} = {} & \text{Present value of material in old line} \\ & + \text{Present value of labor in old line.}\end{aligned}$$

It is evident that the last item in "net increase in investment" cancels the first item in "investment represented by the old line," hence this quantity "present value of material in old line" does not enter into the "total investment represented." Therefore,

$$\begin{aligned}\text{Total investment represented by the new line} = {} & \text{Present value of labor in old line} \\ & + \text{Cost of material in reconstructed line} \\ & + \text{Cost of labor in reconstructed line} \\ & + \text{Cost of labor removing old line.}\end{aligned} \quad (30)$$

In other words, a new line, built to replace an old one which has not outlived its usefulness, represents an investment including the present value of the labor of erecting the old line and the cost of labor in removing the old line in addition to the total cost of material and labor in erecting the new line.

Annual Cost of Reconstructed Line.—The subject of annual cost in general has been discussed in Chap. III and the formation of the general equation in Chap. VI. The application to the special problem of the reconstructed line will be taken up here.

Investment Charges.—In order to obtain the yearly cost chargeable to the reconstructed line which is the quantity which must be used in considering its economy, it is necessary to spread this total investment over a period of time estimated for the new construction. The percentage to be charged annually must include interest, taxes, insurance and depreciation. Ordinarily, at least two different percentages will be used in actual computation. One will apply to labor investment which has no salvage value, and to materials such as poles, crossarms, pins, etc., which have a comparatively short life, and whose scrap value at the end of the useful life is probably about equal to the cost of labor necessary for salvaging. Another figure will be used on such material as bare copper wire which has practically no physical depreciation. Its value at the end of the life of the line is as great as at present, providing prices do not decrease, and the only expense incurred will be the labor cost of salvaging. The market value of copper has varied so widely during the past few years that a great element of uncertainty is introduced into this item. For this discussion, however, it will be assumed that the price of copper will remain constant. For the present both these percentages will be represented by the letter g.

g = per cent interest, taxes, insurance and depreciation chargeable annually against the construction. The expression "$g \times$ any quantity" merely indicates that it is the annual charge against that quantity which is being considered.

Operating Charges.—Another element of annual cost is the operating expense. The chief item under this head will be that of energy loss. If the reconstructed line is of higher voltage or of larger wire size than the old, or if the length is shortened, the energy loss may be materially reduced. This will tend to reduce the annual cost and is an important item when relative economy is to be considered. Here, again, considerable uncer-

tainty is introduced on account of the variation in load factor and load at present and in the future. The value of these items must be selected for the particular problem by careful study of the local conditions, since here, as in any engineering problem, the solution must be based on certain definite assumed values for all variable quantities. The success or failure of the solution will depend upon the good judgment used in such selection. The operating expense also includes the cost of superintending, repairing, patrolling, etc., which will, in most cases, probably not be materially different on the reconstructed line from what it was on the old line. In case the difference is marked this point must be taken into consideration in the final decision.

General Equation for Annual Cost of Reconstructed Line.—A general equation may now be formulated for the annual cost of the reconstructed line by combining these factors.

$$\text{Annual cost of reconstructed line} = g \times \left\{ \begin{array}{l} \text{Present value of labor in old line.} \\ +\text{Cost of material in reconstructed line.} \\ +\text{Cost of labor in reconstructed line.} \\ +\text{Cost of labor removing old line.} \\ +\text{Cost of superintendence, repairs, patrolling, etc.} \\ +\text{Cost of energy losss on recontructed line.} \end{array} \right\} \quad (31)$$

Similarly,

$$\text{The annual cost on the old line} = g \times \left\{ \begin{array}{l} \text{Value of material in old line.} \\ +\text{Value of labor in old line.} \\ +\text{Cost of superintendence, repairs,} \\ \quad \text{patrolling, etc.} \\ +\text{Cost of energy loss on old line.} \end{array} \right\} \quad (32)$$

By proper selection and application of the percentage g, and computation of the various costs involved, in accordance with local conditions, these equations may be applied to any such problems of reconstruction such as determining the relative economy of replacing a present line with one of higher voltage or larger wire, or of shortening a line by rebuilding part of it, as will be shown later.

Utilization of Old Materials in Reconstructed Line.—In the above discussion it was assumed that all the old material would be salvaged. It very often occurs, however, in such cases as an increase in voltage, that the new line may to advantage follow the old route for part of the distance at least, using the old poles and, in some cases, the old crossarms, insulators, etc. The general equation still holds good for this condition if its deriva-

tion is kept in mind and the various items adjusted to fit the conditions. The "cost of material in reconstructed line" must include the "present value" of the old material used, since, it will be remembered, this originally entered the equation as a credit (to be salvaged) and was cancelled out. The "cost of labor in reconstructed line" must be reduced due to the fact that the old material used is already in place but it must also include any labor cost necessary to adapt the old material to the new conditions. The "cost of labor of removing old line" will, of course, be reduced by the amount that would be required for the material not removed. The value of g must also be selected to take into account the fact that the present life of the old material will not be as great as the new and hence the labor necessary in adapting it will have a greater yearly depreciation. With these conditions clearly in mind, however, it is evident that the general equation may be safely applied to any such problem of reconstruction, even when part of the old construction is to be utilized. This would include also the case when a second line was to be added on the same poles.

So far the discussion has been entirely of a general nature. It has been shown, how, in any problems involving reconstruction of a line still serviceable, the investment cost and annual charges pertaining to the old line must be included in the costs chargeable to the new line. These costs will be increased or decreased by the various extra expenditures or savings belonging to the new line. It is now proposed to show how the general method can be applied to specific cases, and how in some cases "short-cuts" may be introduced to greatly simplify the problem.

Economy of Shortening a Line already in Service.—Let us assume, for example, a line several years old which is in good condition and of sufficient capacity to care for the predicted load up to the probable length of its life. Since its construction, however, conditions have so changed that it is now possible to reduce its length considerably by using a more direct route in some places. The question arises as to whether or not it would be economical to rebuild those portions of the line by the shorter routes. In order to determine this it is necessary to compare the annual cost of the present installation with the total annual cost chargeable to the reconstructed installation, if the change is made. These annual costs may be determined by use of the general equations previously developed.

The present annual cost charged to the section of the line under consideration is given by Eq. 32.

The annual cost in case the old line is replaced by the new section would be as shown by Eq. 31.

It would be economical, then, to undertake the new construction only if the annual cost of the reconstructed line, thus computed, is less than that of the old line. The added construction cost must be offset by the reduced energy loss. The limiting case is when the two annual costs are equal:

Annual cost of reconstructed line = Annual cost of old line. It is evident, however, that several of the items in both annual costs are practically the same, so for the purpose of this comparison they may be eliminated. Since the reconstructed section is but a small part of the line as a whole, its useful life must be assumed to be that remaining in the old line. Hence the annual charges against the "present value of labor in old line," which is part of the "annual cost of the reconstructed line," will be practically equal to the annual charges against "cost of labor in old line," which is part of the "annual cost of old line." The cost of "superintendence, repairs, patrolling, etc.," will be practically the same for both, unless the difference in length is large. The equation then reduces to

$$g \times \left\{ \begin{array}{l} \text{Cost of material in reconstructed line} + \\ \text{Cost of labor in reconstructed line} + \\ \text{Cost of labor removing old line} \end{array} \right\} + \text{Cost of energy loss on reconstructed line} = g\,(\text{Cost of material in old line}) + \text{Cost of energy loss on old line} \tag{33}$$

The cost of material and labor in the new line will of course include the cost of any private right-of-way which it is necessary to purchase and of any other extra expense connected with the construction.

A system of symbols will be adopted, conforming to those previously used.

Let L_2 = length in miles of the new section to be built,
L_6 = length in miles of the old section to be replaced,
kw = load in kilowatts,
L_4 = the length in miles of right-of-way to be purchased,
N = number of new corners necessary in reconstructed line,
C_r = cost per mile for right-of-way.

The details of the computation of unit costs used here need not be given as they would apply to only one particular case. The figures used are based on a line at 46,000 volts, single construction, on wooden poles, with No. 0 bare copper wire, energy costing 1 ct. (.01) per kilowatt-hour. The annual cost on such new construction introduced into an old line will vary somewhat with the age of the line as explained above. However, since such a reconstruction would probably not be considered except during the early life of the line, and of course, the shorter the remaining useful life assumed, the greater the salvage value of the new material at the end of that life, this may be assumed to be constant for any age. This assumption would also be favored by the probability that this new section would be used, at least in part, beyond the life of the rest of the line.

The unit costs are then assumed to be as follows:

Annual charges on material and labor in reconstructed line	=	\$360.00 per mile
Annual charges on cost of labor removing old line	=	30.00 per mile
Annual charges on cost of material on old line	=	225.00 per mile
Annual cost of energy loss	=	$\frac{0.69\,kw^2}{10^5}$ per mile
Annual cost of new corner	=	40.00
Annual cost of right-of-way	=	$0.08\,C_r$ per mile

Substituting these values, the equation for determining economy becomes

$$360L_2 + 40N + 0.08C_rL_4 + 30L_6 + \frac{0.69}{10^5}kw^2L_2 = 225L_6 + \frac{0.69}{10^5}kw^2L_6$$

$$165L_2 + 40N + 0.08C_rL_4 = (L_6 - L_2)\left(\frac{0.69}{10^5}kw^2 + 195\right) \qquad (34)$$

There are too many variables to exhibit this equation as a curve. The limiting values, however, can be so expressed. If no right-of-way need be purchased and no new corners are added, the greatest length of a new line, L_2, in order to effect a saving of 1 mile ($L_6 - L_2 = 1$) would be

$$\frac{L_2}{L_6 - L_2} = \frac{\frac{0.69}{10^5}kw^2 + 195}{165} = \frac{0.418}{10^7}kw^2 + 1.18 \qquad (35)$$

This is plotted in curve A (Fig. 26a). This curve shows the greatest length of new construction which could be economically built under the most favorable conditions, that is, normal line construction cost with no corners and no other extra expense for right-of-way, etc., for each mile subtracted thereby from the

total length of the line. If such other extra expense is necessary the value of $\frac{L_2}{L_6 - L_2}$ will be less, as may be seen from the equation (34). Hence this curve may be used as a test. If the length of new line under consideration is greater than that shown by the curve, it will not be economical to build it. If it is less, the exact economy must be determined from the equation (34). For

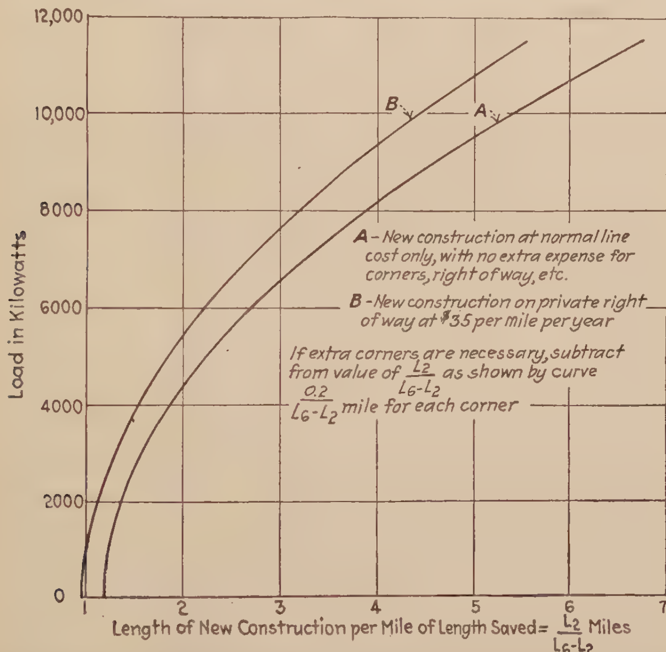

FIG. 26*a*.—Curve showing maximum economical length of new construction, replacing old line, per mile of length eliminated.

example, as a rather extreme case, suppose there are extra corners necessary and the total distance, L_2, will be on private right-of-way costing $35 per mile per year.

$$\frac{L_2}{L_6 - L_2} = \frac{0.345}{10^7} kw^2 + 0.975 - \frac{0.2N}{L_6 - L_2} \tag{36}$$

If the last term is omitted this equation may be plotted as shown by the curve *B*, Fig. 26, and gives the economical values of $\frac{L_2}{L_6 - L_2}$ (new construction per mile saved) when the extra construction cost is fairly large. For extra corners $\frac{0.2}{L_6 - L_2}$ miles

must be subtracted from the value shown by the curve for each additional corner. Of course there might be other extra costs for more poles, higher poles, etc., which have not been especially considered here. These could be included in the same manner however.

The economy for any particular loading on such a line may now be readily discovered from the curves and equations. If, for example, under the conditions assumed, the load is 6,000 kw. for no corners and no cost of right-of-way, according to curve *A*, the new construction could be built for 2.7 miles for each mile saved thereby, or say, 8.1 miles in order to save 3 miles. In case the new construction requires private right-of-way, as per curve *B*, 2.2 miles may be built for each mile saved or 6.6 to save three miles. If two extra corners were necessary, the figure would be 1.8 miles for only one mile saved, or for three miles, 2.067 per mile or 6.2 miles and so forth.

There is thus displayed a tangible method of exhibiting the economy of shortening an old line by reconstruction. For any particular system, of course, the costs must be derived independently. From them, similar curves can be drawn and applied to the solution of such problems.

Economy of Using Part of Old Line Equipment in its Original Location for Reconstructed Line.—Another characteristic problem of this general nature is the following. In building a new line to replace an old one, which is not worn out, it may be possible to use a considerable length of the old line poles or other materials in place. However some shortening of the length of the line might be accomplished by using a more direct route on entirely new construction. How far from the direct route may the line diverge economically in order to use this old line material?

The solution is, again, reached by a comparison of the annual costs on the two routes using the general equation already developed. This comparison, however, may be considerably simplified, as in the preceeding problem, by making use of certain short cuts which reduce the number of terms to be considered. The "present value of labor in old line" is a cost chargeable to the reconstructed line as a whole. It will be the same no matter whether the old material is used in place or not. Hence, it may be omitted in the comparison of economy of the two methods of construction on any section of line where the use of old material is considered. The "cost of labor removing the

old line" is also chargeable to the new line as a whole and it might be somewhat difficult to properly apportion the amount to be charged to any particular section. This may be avoided if it is considered, for the comparative costs, that this item is the same amount for both alternative constructions and hence may be omitted from both. Since, however, some of the old material left in place will reduce the actual amount of this item in the one case, the cost represented by the old materials used must be adjusted by subtracting from their present value the amount which it would have cost to remove them. It is seen that this method is equivalent to charging both alternatives with the proper share of the "cost of removing old line" and then subtracting from one the removal cost eliminated by the use of the old material in place. It is also equivalent to considering that old material in place should be charged at its warehouse value which is its present value as material, if available, less the cost of making it available or removing it to the warehouse. The adjusted annual costs of the reconstructed line on old poles, and of new construction, per mile, may be thus obtained and are convenient figures to use in all such economic comparisons. It must always be borne in mind however that these are adjusted costs and are not the true annual costs chargeable to the line.

The comparison of the annual costs of the two routes may now be made by comparing the values obtained from the expression

g (cost of material and labor (adjusted cost)) + cost of energy loss;—with the proper selection of g, for both alternatives.

The following symbols have been used:

L_1 = length in miles of the old pole line which can be used,

L_2 = length in miles of the alternative new line,

L_3 = length in miles of the new construction necessary to supplement the old pole line, *i.e.*, bringing the line from the new route to the old if necessary,

N = the number of new 90° corner constructions saved by using the more direct route,

kw = load carried in kilowatts,

a = age of old-pole line in years,

C_e = cost of energy per kilowatt hour,

L_4 = length in miles, of right-of-way to be purchased,

C_r = purchase price per mile of right-of-way.

The figures here given are based on an old line on untreated

wood poles, new line on treated poles, voltage of reconstructed line 46,000 volts necessitating new crossarms and insulators throughout, wire No. 0 bare copper on both old and reconstructed lines.

The annual charges obtained under these conditions are as follows: Annual charges on new construction per mile $= \$350.00$

Annual charges on reconstruction on old poles per mile $= \dfrac{3{,}350 - 260a + 3.2a^2}{15 - a}$

Annual charges on energy loss $= \dfrac{69kw^2C_e}{10^5}$

Annual charges on corner construction $= \$38.00$

It is now a question of whether the annual cost on $L_1 + L_3$ is greater or less than on L_2 using the above figures. It would be advantageous to divert to the old pole line when

$$L_1\left(\frac{3{,}350 - 260a + 3.2a^2}{15 - a} + \frac{69kw^2C_e}{10^5}\right) + L_3\left(350 + \frac{69kw^2C_e}{10^5}\right) + 38N < L_2\left(350 + \frac{69kw^2C_e}{10^5}\right)$$

$$L_1 < \frac{(L_2 - L_3)\left(350 + \dfrac{69kw^2C_e}{10^5}\right) - 38N + .08L_4C_r}{\dfrac{3{,}350 - 260a + 3.2a^2}{15 - a} + \dfrac{69kw^2C_e}{10^5}} \tag{37}$$

An example of the use of this equation would be in such a case as that of a line following a private right-of-way instead of a highway thereby eliminating four corners and considerable distance. If we assume the old line to be 5 years old ($a = 5$) and $C_e = .01$ and right-of-way at \$35 per mile per year the equation would become for that particular case with $L_4 = L_2$; $L_3 = 0$; $N = 4$.

$$L_1 < \frac{L_2\left(350 + \dfrac{.69kw^2}{10^5}\right) - 152 + 35L_2}{\dfrac{3{,}350 - 1{,}300 + 80}{10} + \dfrac{.69kw^2}{10^5}}$$

$$L_1 < \frac{L_2\left(385 + \dfrac{.69kw^2}{10^5}\right) - 152}{\dfrac{.69kw^2}{10^5} + 213} \tag{38}$$

which is easily solved for known values of L_1, L_2 and the load and the relative economy thus determined. If L_1 is greater than the

second member, the economy evidently lies in the new route, if less, in the old one. No curve could be plotted which would be of any great value in this instance since there are so many variables in the expression which are fixed for only one particular problem or condition.

Application of Above Method of Choice of Transmission Route.—There is a very useful application of the principles of this last problem in the choice of a route for a transmission line as a whole. When there are several alternative routes a condition similar to the following concrete example is very often encountered. Power was to be transmitted to a distance of approximately 30 miles from the central station. There were three routes available for the new line no one of which had any evident marked advantage over the others. Each one included a different amount of private right-of-way, of old pole line, of new construction, of corners, and of distance over which the construction cost would be excessive on account of high trees and other obstructions. The comparative economy of the three routes was displayed, by use of the adjusted annual costs explained above, and tabulated as follows:

Table 5.—Adjusted Annual Costs

Item	Route *A*		Route *B*		Route *C*	
	Miles		Miles		Miles	
New construction at $350 per mile	21	$ 7,360	19	$ 6,650	22	$ 7,700
Reconstruction on old poles at $213 per mile	8	1,710	12	2,560	6	1,280
Right-of-way at $35 per mile	6	210	4	140	10	350
Corners at $38 each	9	342	7	266	7	266
Energy loss, 4,000 kw. load at $110 per mile	29	3,190	31	3,410	28	3,080
Difficult construction	3	....	2½	....	3	
Total	..	$12,812	..	$13,026	..	$12,676

If is evident that Route *C* is the most economical even though it requires the purchase of more private right-of-way and uses less of the old pole line, since it is somewhat shorter and some corners are eliminated. The extra cost of difficult construction is

rather hard to arrive at without a detailed layout of the line. It can be estimated however. In the above case, the amount of extra construction was so nearly equal in each case that this was not thought necessary.

As has been explained in the previous chapter there are some other features of a line that cannot be included in such a cost analysis and which might have considerable weight in the determination of a route. One route might be more accessible for patrolling and repairs than the others. The proximity to a railroad might affect the construction cost considerably. The shelter afforded the line against severe storms might be a large factor for consideration. These points can however be weighed against the advantage in cost alone and the most advantageous route will usually be evident.

In the foregoing discussion, there has been treated only a few of the many problems involving reconstruction which are encountered by the engineer. However it has been attempted to set forth the underlying principles of the analysis of costs on such problems so that they can be applied to any similar problems with modification to suit the particular case. The curves and figures given herewith are not intended for use under any other conditions than those for which they were derived. They merely serve as an example of how the methods used may be applied to a given case. The outstanding feature, which is recognized after the application of these methods to a few specific problems, especially problems of relative economy, is the great advantage ordinarily gained by shortening a line. As a rule any ordinary extra construction cost is justifiable if the line is noticeably shortened thereby. This, therefore, points to the advantage of greater care in the selection of the routes for new lines in new territory even though at the time it may seem advisable, on account of light load, to keep the construction cost a minimum.

CHAPTER XI

POWER CIRCUITS

PROBLEMS RELATING TO LINES CARRYING POWER LOAD CHIEFLY —VOLTAGE—ECONOMICAL CONDUCTOR SIZE—USE OF TWO LINES IN PLACE OF ONE—DISTRIBUTION OF LOAD OVER SEVERAL LINES

The study of primary lines will be divided into two parts, *i.e.*, the consideration of circuits carrying power chiefly and of those devoted largely to lighting load. It is realized that on most systems, there is no sharp division between these two classes. In most cases, where the power load is comparatively small, power and lighting are both carried on the same lines. When larger power loads come on, the difficulties of regulation usually call for a separation of circuits, even though the load factor on the lines is thereby reduced somewhat. The facts that lighting load requires a closer regulation than power, and that, when large power loads are considered, lighting and power load usually overlap considerably during the heavy loading season, justify such a practice.

On some systems, the power and lighting circuits are kept entirely distinct. On others, where three- or two-phase circuits are used for lighting, small- and medium-sized power loads are taken on the same circuits, while separate lines are run for large power loads. In the first case, it is very often the practice to run a three-phase line to some central feeding point and there separate the phases into individual single-phase circuits for lighting. In the latter case, all branches carrying lighting only are ordinarily single-phase. It appears, therefore, that the problems relating to power only, to power and lighting combined and to lighting only may be very similar up to a certain point, differing only in load factor. On the other hand, if lighting only is considered, the quite definite load factor simplifies the study somewhat. Also the single-phase lines are problems in themselves. This chapter will be devoted to the problems of power circuits.

Kind of Problems Encountered.—The questions arising in connection with power circuits have to do mostly with voltage and conductor size. Such lines are nearly always run on roads, streets, alleys or lot lines and the pole spacing and location is limited by the mechanical strength of the pole, by the arrange-

ment of street and lot lines, by provisions for future extensions, etc. Pole heights are governed by standard practice, by city ordinances and by heights necessary to clear obstructions. Occasionally there may arise questions as to the economy of using private right-of-way instead of the public highway for short distances but such problems are usually small ones and are easily solved by a comparison of the annual cost of the two alternatives.

Voltage.—The voltage for use on power circuits is usually a development from past practice, although it is often found economical to increase the voltage when the load increases beyond a certain amount. The voltages in common use have been pretty well standardized at 2,200, 4,400, 6,600 and 11,000 volts. There is a tendency at present to go even higher for power lines with heavy loads, but such lines partake more of the nature of transmission lines. On any line, the higher the voltage the less the line losses and the larger the loads which can be carried on the line with a given regulation. On the other hand, the cost of insulation of the line and the cost of transformers is increased. The additional precautions which it is necessary for construction men to take in working with a higher voltage is also a factor to be considered. To give a comparison, 6,600-volt transformers can be obtained for about 18 or 20 per cent more than 2,200-volt. Thereby the voltage is multiplied by three, hence the line loss is divided by nine for any given load and wire size. Or, for the same per cent voltage drop and conductor, nine times the load can be carried. Where loads are not heavy, such an increase in capacity may not be desirable as compared with the increased cost of construction. Where heavy loads are handled, however, a voltage of 6,600 or 11,000 may often be found very advantageous. When the problem is one of changing the voltage of a system already in operation to a higher voltage, the cost of making the change must be taken into account. The old line transformers must be disposed of, or the change made gradually, using the old transformers in certain districts until they are worn out. Station transformers and other apparatus, suitable for the higher voltage, must be provided. Often, cables must be replaced with those of higher rating. The increase in annual charges due to all these items must be carefully studied in connection with the value of the increased capacity and reduction in losses achieved, in order to determine the economy of any such alteration. In this connection, the probable increase of

load on the system for some time in the future must be estimated and what further changes will have to be made eventually to care for probable future conditions.

Voltage Drop.—The problem of voltage drop is an important one to consider in connection with power circuits. The allowable regulation at the customer is more or less fixed by considerations of good service or by contract. The substation bus voltage may be kept within certain known limits. The question is then one of whether to serve the customer by a circuit of small or medium-sized conductor with a regulator, or of large-sized conductor without a regulator. The annual cost of the installation as a whole, including cost of energy losses, will be the criterion. On a large system with a steadily increasing load it is often the practice to standardize on one or two conductor sizes. A new line is built of standard size and allowed to operate at low loads unregulated. Load is added from time to time until, when it becomes too heavy, a regulator is added. From a practical standpoint this method has its advantages. A study of the economy of the installation, however, will still be of real advantage in indicating the standard sizes to use, when the lines become loaded beyond the economical limit, etc.

Power-factor Improvement.—The question of power factor is a prominent one at present. Aside from the proposition of inducing the customer to improve his power factor by using that as a basis for rates, there is a further interesting problem for the central station. Poor power factor means increased losses for the same delivered load in kilowatts. There will be, then, a point at which it will be economical to install static or synchronous condensers in order to reduce these losses by improving the power factor. This point can be determined by a comparison of the annual cost of the condenser in place, with the value of the energy conserved, considering also the improvement in regulation.

Most Economical Conductor Size.—The determination of the most economical size of wire for any load, is closely connected with the considerations of voltage and voltage drop, as is evident from the above paragraphs. If the most economical conductor size, considering the line only, is known, however, it serves as a starting point for the further study of the economy of the circuit as a whole, including regulators, etc. An example will be given here of the method of attacking this problem of most economical wire size for power circuits.

Annual Cost Equation.—For a three-phase line the annual cost per 1,000 ft. of line

$$= g \text{ (cost of wire + cost of stringing)}$$
$$+ g \text{ (cost of poles, fixtures, guys, etc.)}$$
$$+ \text{ cost of energy loss.}$$

Where g = per cent interest, taxes, depreciation, etc. (Cost of right-of-way is omitted as it is not always present on such lines, and, in any case, is the same for all sizes of conductor.)

In this case, instead of developing the above equation in terms of the cross-sectional area of conductor and then, by means of the first derivative, determining the most economical size, it has been found more useful to investigate the range of loads for which any given stock wire size is more economical than any other. The method used is as follows:

The annual cost per 1,000 ft. of line for each standard conductor size is obtained in terms of the cost of copper, the load, voltage, power factor, equivalent hours, and cost of energy. This equation is of the form

$$Y = K_1 + K_2C_{cu} + K_3\left(\frac{kw}{E \cos \theta}\right)^2 tC_e \,. \tag{39}$$

Where Y = annual cost per 1,000 ft. of line,
C_{cu} = cost per pound for copper,
kw = load in kilowatts,
E = voltage,
Cos θ = power factor,
t = equivalent hours,
C_e = cost of energy per kilowatt-hour,
K_1, K_2, K_3 = constants.

Combined Equation.—If the equation for any stock size of wire is combined with that of the next adjacent size by equating the annual costs, another equation is obtained which expresses the conditions under which there is no choice in economy between the two. If, for example, in the above equation E, and C_{cu} are fixed, the combined expression would give, for any value of tC_e, the load at which the economy changes from one size of conductor to the next. If such expressions are determined and plotted for No. 6 to No. 4 and for No. 4 to No. 2, for example, the values of $kw/\cos \theta$ between the two curves, for any value of tC_e, indicates the range of loads for which No. 4 wire is more

economical then either adjacent size. For smaller loads, No. 6 is more economical, for larger loads, No. 2 (see Fig. 27).

Value of Constants.—In the above equation (39) the constants K_1 and K_2 depend on wire size, local costs for stringing wire, and local standards of construction and costs for use with each wire size. It may be assumed in this case that the poles will be the same for any size of conductor, and hence their cost will cancel out when the equations for two wire sizes are combined.

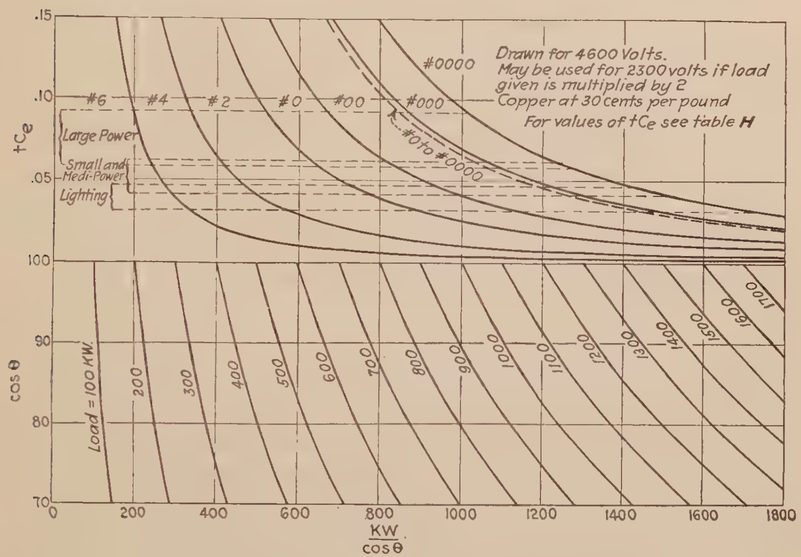

FIG. 27.—Economical wire size for three-phase primary.

The cost of pole fixtures (crossarms, etc) does not increase proportionally with wire size but will be the same for several sizes and then change abruptly for the next larger group. The cost of conductor in place, including incidental material and labor cost of stringing, will follow such an expression as $K_a + K_b\,C_{cu}$, K_a and K_b being separately determined for each size of wire. These constants, properly combined give the values of K_1 and K_2.

Cost of Energy Loss.—The cost of energy loss is determined from the equation: Annual charge for energy loss per 1,000 ft. =

$$3I^2r \times 365 \times t \times \frac{C_e}{1{,}000} = 365{,}000\,rtC_e\left(\frac{kw}{E\cos\theta}\right)^2 \qquad (40)$$

Where r = resistance of conductor per 1,000 ft.

Then K_3 (in Eq. 39) $= 365{,}000\ r$, for any size of conductor.

Numerical Example.—The following equations of total annual cost were obtained in a specific instance, with all constants evaluated.

TABLE 6.—EQUATIONS OF TOTAL ANNUAL COST

Size of Wire	
6	$6.7 + 56.2C_{cu} + 147{,}000\left(\frac{kw}{E\cos\theta}\right)^2 tC_e$
4	$8.41 + 80.0C_{cu} + 92{,}500\left(\frac{kw}{E\cos\theta}\right)^2 tC_e$
2	$11.30 + 126.0C_{cu} + 58{,}100\left(\frac{kw}{E\cos\theta}\right)^2 tC_e$
0	$12.43 + 204.0C_{cu} + 37{,}300\left(\frac{kw}{E\cos\theta}\right)^2 tC_e$
00	$14.62 + 251.0C_{cu} + 29{,}500\left(\frac{kw}{E\cos\theta}\right)^2 tC_e$
000	$16.43 + 313.0C_{cu} + 23{,}400\left(\frac{kw}{E\cos\theta}\right)^2 tC_e$
0000	$17.32 + 382.0C_{cu} + 18{,}600\left(\frac{kw}{E\cos\theta}\right)^2 tC_e$

Equating the expressions for cost for each adjacent pair of wire sizes, the following expressions are obtained.

TABLE 7

Size of Wire		Size of Wire	
6	to	4	$54{,}600\left(\frac{kw}{E\cos\theta}\right)^2 tC_e = 23.8\ C_{cu} + 1.71$
4	to	2	$34{,}400\left(\frac{kw}{E\cos\theta}\right)^2 tC_e = 46.1C_{cu} + 2.91$
2	to	0	$20{,}800\left(\frac{kw}{E\cos\theta}\right)^2 tC_e = 78.0C_{cu} + 1.13$
0	to	00	$7{,}800\left(\frac{kw}{E\cos\theta}\right)^2 tC_e = 47.4C_{cu} + 1.19$
00	to	000	$6{,}100\left(\frac{kw}{E\cos\theta}\right)^2 tC_e = 61.2C_{cu} + 1.81$
000	to	0000	$4{,}800\left(\frac{kw}{E\cos\theta}\right)^2 tC_e = 69.0C_{cu} + 0.89$
0	to	0000	$18{,}700\left(\frac{kw}{E\cos\theta}\right)^2 tC_e = 178.0C_{cu} + 4.89$

Assuming the voltage of 4,600 and two prices for copper, 30 cts. and 20 cts., which represent a good range of values, the equations become

TABLE 8.—FOR 4,600 VOLTS

Size of Wire		Size of Wire	30-ct. Copper	30-ct. Copper
6	to	4	$tC_e = \dfrac{3,430}{(kw/\cos\theta)^2}$	$= \dfrac{2,510}{(kw/\cos\theta)^2}$
4	to	2	$tC_e = \dfrac{10,290}{(kw/\cos\theta)^2}$	$= \dfrac{7,460}{(kw/\cos\theta)^2}$
2	to	0	$tC_e = \dfrac{24,900}{(kw/\cos\theta)^2}$	$= \dfrac{17,100}{(kw/\cos\theta)^2}$
0	to	00	$tC_e = \dfrac{41,800}{(kw/\cos\theta)^2}$	$= \dfrac{28,900}{(kw/\cos\theta)^2}$
00	to	000	$tC_e = \dfrac{70,000}{(kw/\cos\theta)^2}$	$= \dfrac{48,750}{(kw/\cos\theta)^2}$
000	to	0000	$tC_e = \dfrac{95,200}{(kw/\cos\theta)^2}$	$= \dfrac{64,750}{(kw/\cos\theta)^2}$
0	to	0000	$tC_e = \dfrac{66,000}{(kw/\cos\theta)^2}$	$= \dfrac{45,800}{(kw/\cos\theta)^2}$

For 2,300 volts the numerators of the above expressions for 4,600 volts should be divided by 4

$$\left(\text{as No. 6 to No. 4, } tC_e = \frac{857.5}{(kw/\cos\theta)^2}\right)$$

The curves plotted for 4,600 volts can be used for 2,300 volts if the given load for 2,300 volts is *multiplied by* 2 before applying curve.

Plotting Results in Curves.—As was mentioned before in Chap. IX on "Secondary Transmission Lines," the cost of energy losses per kilowatt-hour, C_e, and the equivalent hours, t, are interrelated. For any type of load, such as that on a typical power circuit, the value of C_e corresponding to any value of t, may be determined approximately. For power circuits in heavily loaded districts, such as the industrial areas in a large city, the load factor, equivalent hours, and power factor may be practically the same for nearly all lines. In that case, the problem is simplified and the range of loads for which any wire size is most economical is more simply defined. For the general case of power circuits, however, the load may vary from a single motor to a large manufacturing plant load, from a few hours per week operation to continuous 24 hr. per day. Naturally the load factor will vary through a large range, and the equivalent hours for the same load factor will be different for different loads, depending on the operation.

The above equations therefore have been plotted using tC_e as one coordinate, Figs. 27 and 28. The method of determining

the value of tC_e for any load will be explained later. In order to make the curves applicable to loads of various power factors, the other coordinate was made $\frac{kw}{\cos\theta}$. The computation of $\frac{kw}{\cos\theta}$ for any value of either quantity can be made graphically by use of the curves on the lower half of the figure. The intersection of the curve for any load with the horizontal for the desired power factor gives the value of $\frac{kw}{\cos\theta}$ on the scale below.

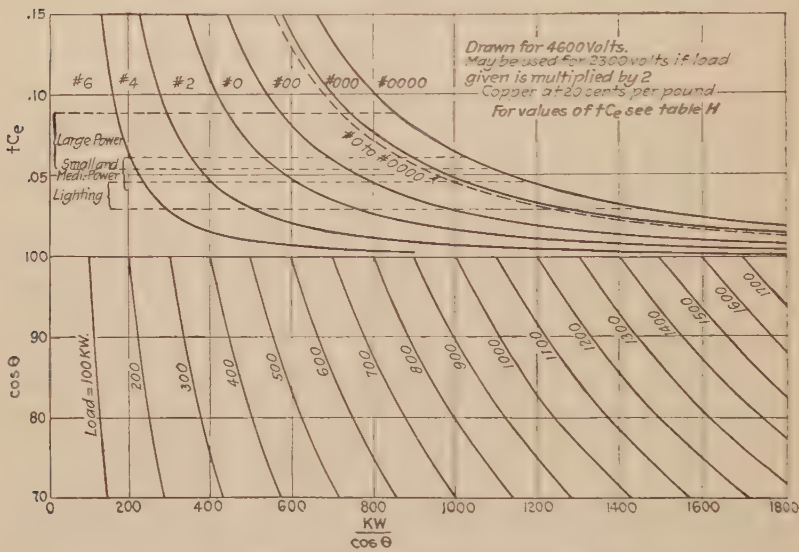

Fig. 28.—Economical wire size for three-phase primary.

Use of Curves.—Since the equations were so developed as to show the points where economy changes from one size of wire to another, it follows that the area between two curves is the locus of all points for which the size of wire shown is most economical. Thus, on Fig. 28, for $\frac{kw}{\cos\theta} = 600$ and $tC_e = 0.06$, No. 0 primary is more economical than No. 2 or No. 00. Similarly it is more economical than No. 2 for all values of tC_e greater than .045 and is more economical than No. 00 for all values of tC_e less than .08. The dotted curve shows the division between No. 0 and No. 0000 which can be used in case No. 00 and No. 000 are not used as standards for such lines. Two sets of curves are given, one for

20 ct. copper and one for 30-ct. Intermediate values can be interpolated.

The use of the curves, then, is as follows:

1. Locate the intersection of the curve for the load in kilowatts with the horizontal of its power factor.

2. Locate the intersection of the ordinate through this point with the proper value of tC_e (scale on left).

3. The area in which this point lies indicates the most economical wire size. The distance of the point from the curve dividing that area from the next adjacent area is an indication of the amount of economical advantage of the one size over the other.

Determination of tC_e.—The proper value of tC_e to use for any load may be determined as follows: It was shown in Chap. V in the discussion of equivalent hours, that the value of equivalent hours corresponding to any load factor may vary between certain limits. The upper limit (load factor $\times$ 24) would be correct only for a load, such as a single motor, which has a constant value for its whole time of operation. The minimum value would be (load factor)$^2 \times 24$ for a load with a momentary peak and the remainder of the day's curve flat. With power loads, it is probable that t varies from somewhere near the first quantity for small loads, such as one or two motors, to somewhere near the average between the two for large loads with a number of motors not running simultaneously. Probably, for most loads encountered, t will be nearer the latter figure.

The limits of t would be as follows:

Table 9

Load factor	Continuous load	Momentary peak	Average
0	0	0	0
.10	2.4	.24	1.32
.20	4.8	.96	2.88
.30	7.2	2.16	4.68
.40	9.6	3.84	6.72
.50	12.0	6.0	9.00
.60	14.4	8.64	11.52
.70	16.8	11.76	14.28
.80	19.2	15.36	17.28
.90	21.6	19.44	20.52
1.00	24.0	24.00	24.00

An approximate determination of the variation of the cost of energy losses with load factor can be made quite easily as indicated in "Appendix *A*." Of course, if more accurate cost figures have been determined they are preferable. The following indicates such a characteristic variation.

TABLE 10

LOAD FACTOR	COST OF ENERGY LOSS PER KILOWATT-HOUR
.10	.0278
.20	.0184
.30	.0150
.40	.0133
.50	.0121
.60	.0113
.70	.0107
.80	.0101
.90	.0096
1.00	.0092

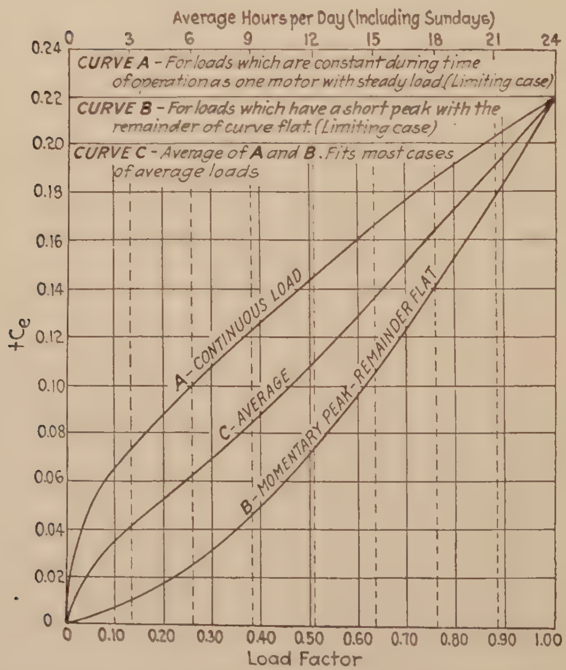

FIG. 29.—Curves showing values of tC_2 for power loads.

Using these figures in connection with the table given above the curves shown on Fig. 29 are plotted which show the values

of tC_e corresponding to any load factor. A second scale is shown at the bottom giving the average number of hours per day of peak operation, corresponding to any load factor, which is useful, especially in connection with small loads. If the load factor of a load is known and the approximate shape of its typical curve, the value of tC_e may be selected. For a single motor averaging 3 hr. a day, for example, tC_e would be about .072. For a larger load with a definite peak and an average load curve, with a load factor of 30 per cent, the value of tC_e would be somewhere near .08. In general it may be said that:

Table 11

	Load Factor	Load Curves	$t\,C_e$
Small power	0 to 20	Continuous to average	0.04 to 0.06
Medium power	10 to 30	Near average	0.04 to 0.07
Large power	20 to 40	Near average	0.05 to 0.09

These figures are indicated on the curves, Figs. 27, 28, and by the brackets on the left. For lines carrying lighting only, the value of tC_e would be between .03 and .05, being nearer the former for residence lighting only, and approaching the latter figure for heavy store lighting, etc., with a fair average of about .04.

For a load which combines power and lighting the value of tC_e will depend on the proportion of each. Where the power predominates, the lighting load will have the effect of increasing the load factor. Where lighting predominates the power load will have the same effect. In either case a higher value of tC_e should be used than would be assumed for the predominating type of load alone. The amount of increase must be estimated from a consideration of the probable load factor for the particular case.

Similar Curves Should be Derived Locally.—Curves such as those illustrated here, may be worked out for any system, using local cost figures. (The examples given here must not be considered applicable to any but the system for which they were derived.) By their use the most economical wire size for any load may be determined and, from that point on, the problem becomes one of obtaining proper regulation in the most economical way.

Power Circuits with t Constant.—As was indicated above, for power circuits in heavily loaded manufacturing districts where

there is considerable diversity of load on each line, but the general characteristics of all loads are somewhat similar, the value of equivalent hours will be nearly the same for all lines. If an average value of t is determined, the problem of economical conductor size may be simplified and it is possible to study the effect of variations in the cost of copper and of energy to better advantage.

Determination of t.—The determination of average equivalent hours for a number of circuits was explained in Chap. V with an example of lighting circuits. For power circuits the method is similar. In a specific instance, bi-monthly curves were taken for a number of power circuits for 13 months and the equivalent hours for each such curve in terms of the curve's peak was determined. This was assumed to be the average for the half month covered. Each of these figures was reduced to a value of equivalent hours in terms of the year's peak by multiplying it by the square of the ratio between the peak for the day for which the figure was derived and the year's peak. The 26 values thus obtained were averaged and the result was assumed to be a fairly accurate figure for the equivalent hours for the whole year, considering days of operation only, exclusive of Sundays and holidays. In the example taken, the value of equivalent hours was found to be 9.69. This is high for ordinary purposes, being obtained at a time of high production during the war period, but will serve as an illustration of the method.

Equations for Annual Cost.—The formulas for annual cost with any size of conductor were altered somewhat for convenience, and to bring out another method of representing the results. It was assumed, as an approximation, that all charges on construction, which are not proportional to the cost of copper, are the same for all sizes, and hence, cancel out when the equations for two sizes are combined. The error thus introduced is small for any two sizes near together, such as No. 6 and No. 4, but becomes greater for such combinations as No. 0 and No. 0000. In any case, however, it can be kept in mind and compensated for when using the resulting curves.

Then, the annual charge per 1,000 ft. on the above basis

$$Y = \frac{g}{100}(3{,}000 w_1 C_{cu}) + I^2 \times \frac{\rho}{A_1} \times 3{,}000 \times t \times 300 \times \frac{C_e}{1{,}000}$$

$$= 30 g w_1 C_{cu} + 900 I^2 \frac{\rho}{A_1} t C_e \qquad (41)$$

for a conductor weighting w_1 pounds per foot and of cross-sectional area A_1, using 300 working days per year to correspond to t as derived above.

If this expression is combined with a similar one for a conductor of weight w_2 and area A_2, the resulting equation may be reduced to

$$C_{cu} = \frac{30\rho(A_1 - A_2)}{gA_1A_2(w_1 - w_2)} I^2t \tag{42}$$

If ρ, t, and g are considered constant and known for any pair of conductor sizes, this equation may be reduced to

$$\frac{C_{cu}}{C_e} = K_4I^2 \tag{43}$$

Curves for Economical Wire Size.—The curves shown in Fig. 30 have been plotted from this equation. Their use is

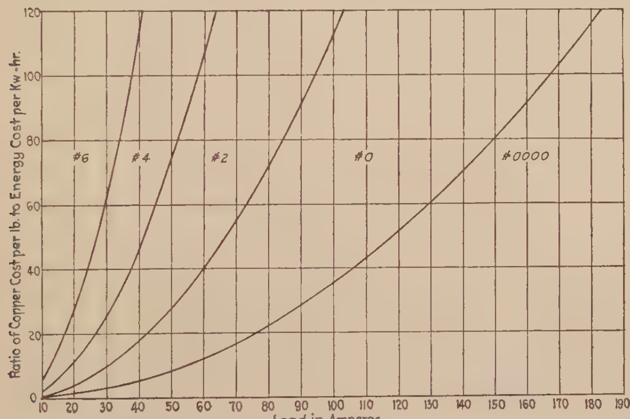

FIG. 30.—Economical wire size for any ratio of copper cost to energy cost for 3ϕ power circuits, equivalent hours = 9.69.

similar to that of Figs. 27 and 28, the areas between the curves belonging to the conductor sizes indicated. It is clearly evident that, as the ratio between copper price and energy cost increases, by copper price increasing or energy cost decreasing, the smaller wire becomes more economical. As was explained above, in using the curve between No. 0 and No. 0000, the excess cost of stringing the larger size must be kept in mind. This would have the same effect as an increase in price of copper, *i.e.*, to raise the curve somewhat.

Use of Two Lines in Place of One.—The study of economical conductor size may be further extended, in a similar manner, to

the consideration of the economy of the use of two lines in place of one. Assuming that, for mechanical reasons, No. 0000 wire is the largest size which is used as a standard on a given overhead system, there is a load at which it becomes economical to use more than one No. 0000 circuit. Another circuit of any size of conductor might be added, according to the load. For simplicity, the problem here will be limited to that of discovering at what load two No. 0000 lines are more economical than one, assuming the same conditions of load as in the previous example, with t constant.

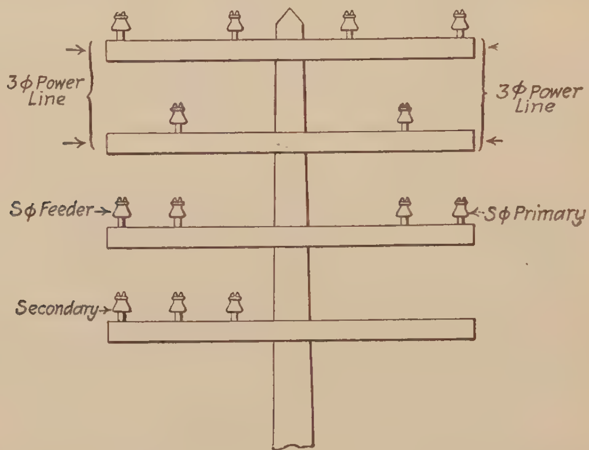

Fig. 31.—Arrangement of wires showing average line conditions.

In this case, the charges on construction cannot be assumed to be the same for both conditions since twice as much pole space is occupied by two circuits as by one. In some cases, this would simply cause the addition of an extra crossarm. It very often occurs, however, that, in districts where such large loads are found, the poles are heavily loaded and all available pin positions are valuable. In such a case, the circuit should be charged with that proportion of the total cost of poles and fixtures equal to the proportion of the total available pole space which it occupies. If, for example, such an arrangement as shown on Fig. 31 is assumed to represent average line conditions, one of the three-phase lines occupies one-fourth the pole space. If the average cost of a pole and four crossarms with other fittings is $60 the cost of pole space for the line in question is $15 per pole, or about $150 per 1,000 ft.

If C_p = cost of pole space per 1,000 ft.
and C_c = cost of stringing, conductor, insulators and pins per 1,000 ft.

The total annual cost for one circuit

$$Y_1 = \frac{g}{100}(3{,}000wC_{cu} + C_p + C_c) + 900\, I^2 \frac{\rho}{A} tC_e \tag{44}$$

and for two circuits

$$Y_2 = \frac{2g}{100}(3{,}000wC_{cu} + C_p + C_c) + 2 \times 900 \left(\frac{I}{2}\right)^2 \frac{\rho}{A} tC_e \tag{45}$$

The investment cost is doubled while the energy loss is halved. Equating Y_1 and Y_2

$$\frac{g}{100}(3{,}000wC_{cu} + C_p + C_c) = 450I^2 \frac{\rho}{A} tC_e \tag{46}$$

In order to put the curves in the same form as those for single lines, this equation must be reduced to an evaluation for $\frac{C_{cu}}{C_e}$

$$\text{If } k = 1 + \frac{C_p + C_c}{3{,}000wC_{cu}}$$

$$\frac{g}{100}(3{,}000wC_{cu}k) = 450I^2 \frac{\rho}{A} tC_e$$

$$\frac{C_{cu}}{C_e} = \frac{15\rho t}{gwkA} I^2 \tag{47}$$

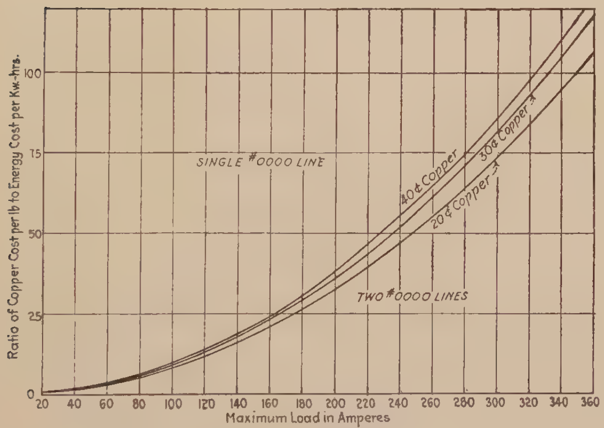

Fig. 32.—Economical circuits for any ratio of copper cost to energy cost for three-phase power.

If ρ, t, g, w and A are constant and k is evaluated for several values of C_{cu}, this equation can be plotted in a series of curves as

shown on Fig. 32, one curve for each value of C_{cu}. This could have been added to Fig. 30, if desired, as it is of the same form.

Improvement of Regulation by Two Circuits.—An interesting point arises in connection with the use of two circuits instead of one when a low power factor is encountered. While the resistance drop of a conductor decreases proportionally with the increase in size, the inductance drop decreases slowly. Hence, for power factors below a certain value, little improvement in regulation is accomplished by increasing the size of conductor. On the other hand, the addition of a second circuit materially reduces the inductance drop and hence the regulation.

As a concrete example, compare the voltage drop on a No. 0000 circuit carrying 3,000 kw., 7,500 ft., with two No. 0 circuits, both at 4,600 volts, three-phase, with 28-in. spacing. The resistance of No. 0 wire being about twice that of No. 0000, the equivalent resistance of both installations is about the same. The power loss and voltage drop, at different power factors, as figured by the charts in Chap. VII, are as follows:

TABLE 12

Power factor	Power loss		Voltage drop	
	One No. 0000	Two No. 0	One No. 0000	Two No. 0
Per cent	Per cent		Per cent	
50	14.6	14.25	18.05	10.97
85	5.03	4.87	8.78	6.04
95	4.05	3.97	6.64	5.05

It is very evident, from the above, that, at a low power factor, considerable advantage in regulation is gained by using two circuits of small conductor rather than one circuit of twice the size. This would be economical if the improvement in regulation is worth more than the increased cost of construction. At high power factors, the advantage disappears since the voltage drop is practically equal to the resistance drop.

Economical Distribution of Load over Several Lines.—One more typical problem will be included in this discussion of power circuits. In cases where there are several different lines, of different lengths and conductor size, feeding a large load, it is

desirable to determine the most economical division of the load among those lines. For simplicity consider two circuits only.

If I = the total load current,

I_a = the economical current on line a,

I_b = the economical current on line b,

$I = I_a + I_b$ (approximately).

Since the lines are already in place, the annual charges on construction will be constant for each

K_a = annual charges on construction for line a.

K_b = annual charges on construction for line b.

The annual cost of energy loss on each circuit will vary with I^2 and C_e, and with the resistance, t being fixed.

$K_cI_a^2C_eR_a$ = cost of energy loss in line a.

$K_cI_b^2C_eR_b$ = cost of energy loss in line b.

Then the total annual charges on line a,

$$Y_a = K_a + K_cI_a^2C_eR_a \tag{48}$$

and on line b,

$$Y_b = K_b + K_cI_b^2C_eR_b \tag{49}$$

and on the total installation,

$$Y = Y_a + Y_b = K_a + K_b + K_cC_e(I_a^2R_a + I_b^2R_b)$$

Substituting for I_b its value $I - I_a$,

$$Y = K_a + K_b + K_cC_e(I_a^2R_a + R_b(I^2 - 2I_aI + I_a^2) \tag{50}$$

and the annual cost per ampere carried

$$\frac{Y}{I} = \frac{K_a + K_b}{I} + K_cC_e\left(R_a\frac{I_a^2}{I} + R_b\left(I - 2I_a + \frac{I_a^2}{I}\right)\right) \tag{51}$$

The condition of maximum economy is reached when $\frac{Y}{I}$ becomes a minimum. The value of I_a to accomplish this may be determined by taking the first derivative of $\frac{Y}{I}$ with respect to I_a, since I is constant

$$\frac{dY/I}{dI_a} = K_cC_e\left(\frac{2R_a}{I}I_a - 2R_b + \frac{2R_b}{I}I_a\right) = 0$$

$$I_a = \frac{R_b}{R_a + R_b}I \tag{52}$$

Similarly

$$I_b = \frac{R_a}{R_a + R_b}I \tag{53}$$

and

$$I_aR_a = I_bR_b \tag{54}$$

In case more than two lines are considered, two or more lines can be represented together as one equivalent circuit and the same mathematics apply.

From the above, therefore, the rule can be formulated that: The most economical distribution of a load over several circuits is effected by making the load on each circuit inversely proportional to its resistance, *i.e.*, by making the IR drop equal on all lines.

This is what might have been expected if it is considered that the total annual charges on construction will be the same, regardless of the division of the load, and hence, the object is to make the total energy losses a minimum. The accompanying Fig. 33 indicates how the total cost of energy loss on two circuits

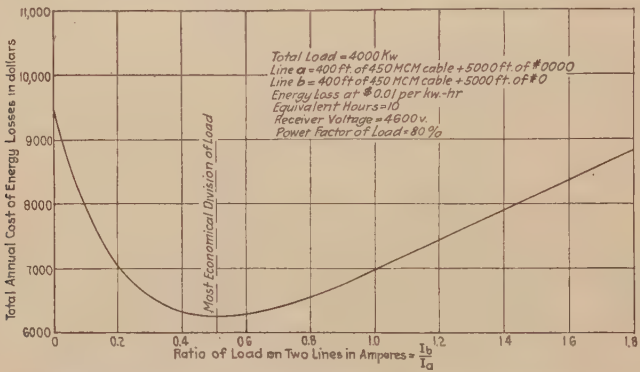

FIG. 33.—Total annual cost of energy losses for various methods of dividing a load between two lines.

with different divisions of load can be displayed graphically. The point of minimum cost is clearly defined.

Division of Load Between Lines in Parallel.—For direct-current circuits the load would naturally divide itself in practically the most economical proportions if the lines are paralleled. For alternating current, however, the inductive reactance has an effect, and the natural division of current would be somewhat different. The currents would divide so that the ratio of the current in any circuit to the load current is the same as the ratio of the admittance of that circuit to the combined equivalent admittances of all the circuits. For example with a No. 0 and a No. 0000 overhead circuit, of the same length, spacing, etc.

$$\text{For No. 0, } R_0 = .539 \text{ ohm per mile}$$
$$\text{No. 0000, } R_{0000} = .269 \text{ ohm per mile}$$

The most economical division of current would then be

$$I_0 = \frac{0.269}{0.539 + 0.269} I = 0.333I \text{ or } 1/3 \text{ the total current}$$

$$I_{0000} = \frac{0.539}{0.539 + 0.269} I = 0.667I \text{ or } 2/3 \text{ the total current}$$

The natural division, if the circuits were paralleled would be

$$I_0 = \frac{Y_0}{Y_{total}} I = 0.444I \text{ or 44.4 per cent of the load current}$$

$$I_{0000} = \frac{Y_{0000}}{Y_{total}} I = 0.566I \text{ or 56.6 per cent of the load current}$$

The fact that the arithmetical sum of I_0 and I_{0000} is not equal to I is due to the fact that they are not in phase with each other, nor with I.

Large Economies Possible on Power Circuits.—The field of study of the economics of power circuits will be found quite extensive and very fruitful of real results. Such circuits in manufacturing districts, usually handle loads many times as large as those on the ordinary lighting circuits, single loads amounting to several thousand kilowatts in some places. The load factor is also comparatively high in most cases. The cost of pole space, on the other hand, is often considerably less per circuit on account of the fact that power circuits are usually run on poles which would be set for lighting circuits in any case. It is evident, therefore, that the saving of a few per cent in line loss on such a line may mean the saving of a considerable sum of money during the year. Hence, the operation under the most economical conditions possible is probably productive of more real saving in money than on any other part of the system. A study of the economics of any type of installation is always beneficial, but in the case of power circuits it is most imperative. The problems exemplified in this chapter indicate the type of question of this kind which will arise most often and illustrate methods which have been found useful for attacking their solution.

CHAPTER XII

LIGHTING CIRCUITS

ECONOMICAL STUDIES ON CIRCUITS CARRYING LIGHTING ONLY—PREDICTION OF LOAD—CONDUCTOR SIZE—INCREASING CAPACITY OF OVERLOADED SYSTEMS

The preceding chapter indicated that in general there were three classes of primary circuits, those carrying lighting load only, those carrying power loads only and those carrying loads made up of a combination of the two. It is here planned to deal with circuits carrying lighting load only—particularly in reference to residence lighting.

The problems encountered in general can be divided into three classes. In the first class are those pertaining to the design of a new system to handle a given or predicted load, such as would be found where it is planned to build a distribution system in a town where no electric service has been furnished before. In this case we meet a problem somewhat similar in characteristics to the one encountered under "backbone" transmission lines. Here our purpose is to design the most economical system possible with few of the factors of design previously established. The only limiting features would be accepted practice, equipment obtainable, and the general knowledge of the subject as recorded in other work of the same class. The problem would therefore resolve itself into one of compiling costs on materials and labor, and working out, as pointed out previously, comparative costs for several different alternative designs using different voltages, different types of primaries, such as single-phase, three-phase, four-wire, etc. The method of determining an economical conductor size will be discussed later on in this chapter. It is a matter of finding the lowest annual charges (investment and energy losses) within the limits of the quality of service desired.

The second class of problems would be that of operating and extending an existing system of primary lighting lines in the most economical manner. This class of problems would be the most commonly encountered in practice. Here certain limitations are found prescribed by the characteristics of the system at hand which would, in general, prevent any considerable change from the practice laid out at the inception of the project.

However, it is advisable to be prepared always to contemplate the possibility of a radical change in such a system. This brings in the third class of problems. They call for a study of the advisability of making such changes as raising the voltage, changing from $S\ \phi$ to $3\ \phi$, three-wire or four-wire, etc.

Predicting Load on Residence Lighting Circuits.—The problem of predicting load on circuits does not involve economics. However, a correct estimate of what should be expected in the next following years is essential as a basis for economical design.

If the demands for electric service which are made each year are to be met economically, they must be anticipated far enough in advance to enable provisions for serving to be made them when it is most economical to do so. About the only way to make intelligent estimates of future requirements is to couple an analysis of past rates of load increase with a far-sighted judgment which will take into account the effect on future conditions of the past rate of growth.

Predicting the load on circuits carrying a load consisting of residence and store lighting is usually simplified by the fact that the growth is relatively constant and that the other factors that come in to modify the estimates can be analyzed and allowance can be made for them.

In the following it will be shown how an analysis of the conditions causing the increase has been attempted for a number of single-phase circuits fed by 200,000-circ. mil. underground cables and No. 0 or No. 0000 overhead wires.

The data available for such a study consist of a set of curves, one for each circuit, showing the monthly maxima over a period of 4 years. These curves are not all of the same outline, yet there are certain characteristics appertaining to all which could be expressed in one curve to be considered typical. A typical curve would serve as a fairly firm foundation on which to base estimates for the future.

The changes in circuit loads, shown by an inspection of the monthly maxima, are caused by a number of easily defined factors. The first is a seasonal condition due to the change in the length of days and the result of cold weather keeping people indoors; the second is an increase in the use of electricity by each customer, caused by the greater appreciation of the uses of electrical energy in the home; the third is due to increase of population within the boundaries of each circuit; the fourth is due

to the embracing of new territory by the circuit or to the loss of old territory caused by switching of load on account of overload or other operating necessities.

The first two factors are constants in their own particular sense—that is to say, the rate of increase due to these factors is a fairly constant value—and the third is partly so, for it may be subdivided into two, the factor of normal increase in population and that of abnormal increase. Normal increase will be encountered in practically every section of the city, abnormal only in sections not yet closely built up and to which people are attracted by real estate activity or the circumstances of industrial development. The fourth factor is a result of the preceding three and must be considered as indefinite, as it appears both for and against increases of load. It is a factor, however, for which correction may be made, since its use lies within the control of the operating company.

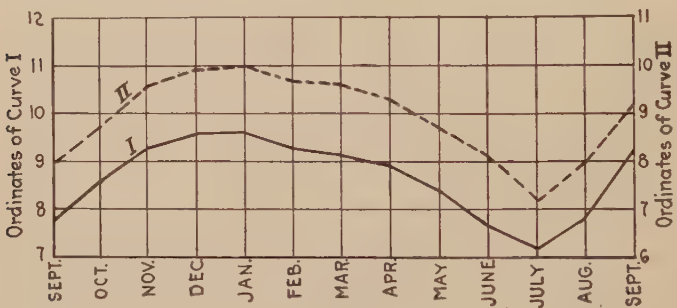

FIG. 34.—Typical curves to show variation in monthly maxima.

The ordinates of curve I, Fig. 34 are proportionate to the sums of the maximum monthly loads on the circuits, averaged over 3 years. It was considered that the indeterminate factor of changes in territory would not affect the curve, inasmuch as they appeared in different circuits at different times of the year and their effect would be reduced to a comparatively small value with an average equal effect on all the ordinates of the curve, in view of the long period of time covered. This assumption is borne out by the comparison of curve I with curve II. This last curve covers a period of only one year—and was corrected for changes in territory. The corrections were made by deducting from the totals of each month the amount representing the load in the same territory appearing in two circuits in the same

month. This condition of duplication of load is due to the fact that when a change in boundary for the relief of any circuit is made, the load on the section cut off appears in the maxima of both the circuits relieved and relieving. It would be possible naturally to take a well-built-up district including several circuits and establish a curve showing the growth, per year, of that nature of load, leaving the new circuits, with abnormal growth, for a special study. This refinement is hardly necessary, as the percentage of growth would not be much diminished. In the method used here there is introduced a small factor of safety.

In order to make the curve more easily applicable it has been reduced to a table of percentages. Table 13, in which the load for each month in the year appears as a percentage of every other month in the year. It is possible by the use of this table to take the load on any circuit for any given month and predict on that circuit the load for any future date, always considering that the boundaries of the circuit remain unchanged and that there are no particular conditions in that circuit which will cause an abnormal increase or decrease in the load.

It is not the intention that this curve, or the percentage table developed from it, shall figure as an absolute method for estimating future loads, but merely as a basis on which are to be imposed the particular conditions obtaining for each territory under consideration. The table contains no allowances for abnormal conditions, and the results derived from it may, in some cases, have to be considerably modified by such conditions as changing of circuit boundaries, rapid increase in rate of settlement, and others. However, the table is sufficiently accurate to serve as a basis in estimates on the necessity for future work, unless the speed of growth of the city is greatly diminished or increased from the average rate maintained for the years considered.

The table gives the load on any circuit for any month in terms of any other month, and by correct selection of factors it is possible to predict with reasonable accuracy the future maximum load in amperes on any circuit if a present or previous reading on that circuit is given. For example, the November load on a circuit is 223 amp., and it is desired to estimate its load for January, two years later. Taking the November load as 1 in the table, the load for the following September is .993 of the

November load; then taking this September load as 1, the load for the following January is 1.23 of this calculated September load. In this case, therefore, the January maximum load of this circuit will be $223 \times .993 \times 1.23 = 272$ amp. This table should be read downward for predicting future load, upward if past loads are to be determined.

Table 13.—Ratio of Each Month's Load to the Total Load for the Remaining Months of the Year

	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.
Sept....	1.000	0.892	0.825	0.815	0.812	0.831	0.836	0.862	0.911	1.003	1.098	0.988	0.831
Oct.....	1.113	1.000	0.913	0.907	0.904	0.925	0.931	0.958	1.013	1.117	1.223	1.100	0.925
Nov....	1.210	1.087	1.000	0.986	0.983	1.006	1.012	1.042	1.102	1.216	1.330	1.196	1.006
Dec....	1.226	1.101	1.012	1.000	0.996	1.018	1.026	1.057	1.117	1.231	1.348	1.212	1.018
Jan.....	1.230	1.105	1.015	1.003	1.000	1.022	1.029	1.061	1.121	1.236	1.352	1.216	1.022
Feb.....	1.203	1.081	0.993	0.980	0.977	1.000	1.006	1.037	1.096	1.208	1.332	1.188	1.000
Mar....	1.195	1.073	0.986	0.974	0.972	0.993	1.000	1.030	1.088	1.200	1.312	1.182	0.993
Apr	1.159	1.041	0.957	0.945	0.943	0.964	0.970	1.000	1.056	1.163	1.273	1.146	0.964
May ...	1.096	0.986	0.906	0.895	0.892	0.912	0.918	0.946	1.000	1.100	1.204	1.083	0.912
June...	0.995	0.894	0.822	0.812	0.809	0.827	0.833	0.858	0.906	1.000	1.093	0.984	0.827
July....	0.910	0.818	0.752	0.742	0.739	0.756	0.761	0.784	0.829	0.914	1.000	0.900	0.756
Aug....	1.010	0.908	0.835	0.825	0.822	0.841	0.846	0.872	0.922	1.014	1.110	1.000	0.841
Sept....	1.203	1.081	0.993	0.980	0.977	1.000	1.006	1.037	1.096	1.208	1.322	1.188	1.000

Having established a table for predicting loads it is applied to the planning of the necessary new equipment required to take care of the expected loads. It will serve not only as a basis for designing new overhead or underground feeders, but also for substation, transmission and power line requirements. It is evident that proper prediction of load is of great importance in the work of economics of distribution as it affects the growth of the system as a whole. Proper care in compilation of data and considerable study of local conditions will be well repaid in bringing about the possible economies of the system.

Economical Wire Size for Single-phase Lighting Primaries.—In extending an existing system the problem of the proper size for lighting primaries will present itself. Loads to be handled will be estimated as shown above. A method of obtaining the equivalent hours for those loads was given in Chap. V. With these two factors known we can proceed, as an example, to determine the economical wire size for single-phase, 4,600-volt primaries. In the preceding chapter on power circuits a complete analysis of a

similar problem was given. Here it is proposed briefly to give the equations and the constants that apply particularly to lighting circuits.

For single-phase lines the cost of conductor will be two-thirds that for three-phase. The cost for pole fixtures will be more than two-thirds on account of the fact that the crossarm cost is included.

The following equations were obtained in a specific instance.

TABLE 14.—EQUATIONS OF TOTAL ANNUAL COST

Size of Wire	
6	$5.70 + 37.5C_{cu} + 294,000 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e$
4	$6.17 + 53.3C_{cu} + 185,000 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e$
2	$8.10 + 84.0C_{cu} + 116,200 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e$
0	$8.86 + 136.0C_{cu} + 74,600 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e$
00	$10.58 + 167.4C_{cu} + 59,000 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e$
000	$11.80 + 208.5C_{cu} + 46,800 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e$
0000	$12.39 + 255.0C_{cu} + 37,200 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e$

Equating the expressions for cost for each adjacent pair of wire sizes the following equations are obtained.

TABLE 15

Size of Wire		Size of Wire	
6	to	4	$109,000 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e = 15.8\, C_{cu} + 0.47$
4	to	2	$68,800 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e = 30.7C_{cu} + 1.93$
2	to	0	$41,600 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e = 52.0C_{cu} + 0.76$
0	to	00	$15,600 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e = 31.4C_{cu} + 1.72$
00	to	000	$12,200 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e = 41.1C_{cu} + 1.22$
000	to	0000	$9,600 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e = 46.5C_{cu} + 0.59$
0	to	0000	$37,400 \left(\frac{kw}{E \cos \theta}\right)^2 tC_e = 119.0\, C_{cu} + 3.53$

Assuming the voltage at 4,600 volts and copper at 30 cts. and 20 cts. per pound, the equations become

TABLE 16.—FOR 4,600 VOLTS

Size of Wire		Size of Wire		30-ct. Copper		20-ct. Copper
6	to	4	$tC_e =$	$\dfrac{1,010}{(kw/\cos\theta)^2}$	=	$\dfrac{705}{(kw/\cos\theta)^2}$
4	to	2	$tC_e =$	$\dfrac{3,420}{(kw/\cos\theta)^2}$	=	$\dfrac{2,480}{(kw/\cos\theta)^2}$
2	to	0	$tC_e =$	$\dfrac{8,320}{(kw/\cos\theta)^2}$	=	$\dfrac{5,670}{(kw/\cos\theta)^2}$
0	to	00	$tC_e =$	$\dfrac{15,100}{(kw/\cos\theta)^2}$	=	$\dfrac{10,850}{(kw/\cos\theta)^2}$
00	to	000	$tC_e =$	$\dfrac{23,500}{(kw/\cos\theta)^2}$	=	$\dfrac{16,380}{(kw/\cos\theta)^2}$
000	to	0000	$tC_e =$	$\dfrac{32,100}{(kw/\cos\theta)^2}$	=	$\dfrac{21,800}{(kw/\cos\theta)^2}$
0	to	0000	$tC_e =$	$\dfrac{22,200}{(kw/\cos\theta)^2}$	=	$\dfrac{15,470}{(kw/\cos\theta)^2}$

For 2,300 volts the same formulas can be used if the load in kw is multiplied by two.

The factor tC_e still remains to be evaluated in order to plot the curves. In the case of lighting circuits the load factor can be taken as practically a constant and the shape of the load curve can be also assumed to remain uniform. Hence a simple value for tC_e can be used instead of keeping it a variable in the equation, as was done with power circuits. This factor, as used in Fig. 35, was assumed as .04. The equivalent hours for lighting were determined previously. The cost of energy combined with this was found to give the above figure as an average.

The curve is plotted for two values of the cost of copper (20 cts. and 30 cts.). The area in which the load ordinate intersects the curve for copper cost indicates the wire size to be used. The economical advantage of the size given is indicated by the distance to the adjacent areas. This method of exhibiting results has the advantage of showing graphically the limits between which one can work and will show readily the effect of an increase or decrease of load.

Use of Regulators.—In connection with the determination of the most economical wire size for a lighting circuit it is sometimes necessary to include a consideration of the cost of a regulator. As with power circuits, it may be desirable to determine whether

it is more economical to use large conductors without a regulator than smaller conductors with a regulator. The annual charges on investment and losses for the regulator must be included with those of the line in this case. With lighting circuits, however, it is usually more necessary to have good regulation than with power circuits to prevent fluctuation in the illumination. Hence it is often preferable to use a regulator in any case, regardless of the exact economy, in order to keep the voltage at or near the center of the load as nearly constant as possible.

Other Problems on Lighting Circuits.—With lighting circuits

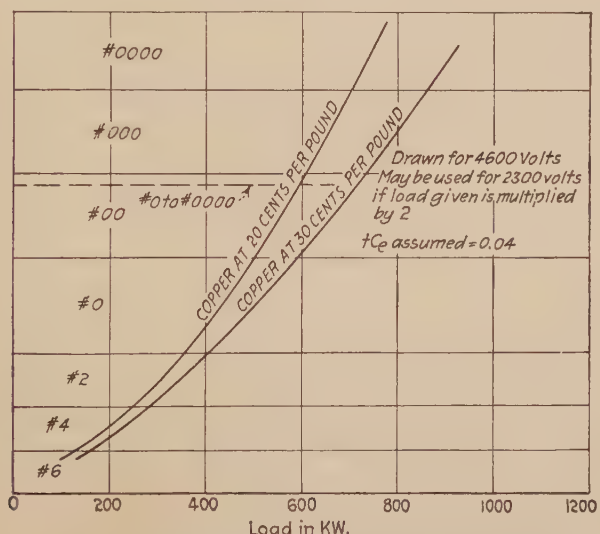

FIG. 35.—Economical wire size for single-phase primaries.

there appear also some of the same problems met with in the preceding chapters, such as the economical load which can be carried on a line already in place, when an additional circuit should be installed or the wire size increased, etc. As the solution is not essentially different from that for other types of lines, these questions need not be discussed in detail here.

Increasing Capacity of an Overloaded System.—It has been the case in many, if not all, large cities, and also in a number of small communities, that the systems of lighting circuits, which a few years ago were apparently perfectly satisfactory for a long time to come, have become inadequate on account of the large increase in the utilization of electricity and in view of the prospec-

tive further increases in the future. This is especially true where comparatively low primary voltages are in use, such as 1,100 or 2,300 volts, and where single-phase circuits, running from the usual type of main sub-stations, is the practice. The change to be made in such a system is a problem warranting the most careful consideration. It is quite probable that future developments may, in a few years, make any provision, which could now be made in the light of present conditions, again inadequate. This cannot be foreseen as yet however. The best that can be done is to provide for an increase at the same rate as at present for a reasonable number of years, leaving, if possible, a good opportunity for further changes at the end of that time, to care for the unforeseen developments. Several methods of providing for this increase in capacity have been tried in different cities with good success. The principal ones will be noted here with brief notes on the advantages claimed and some factors which affect the making of such changes.

Probably the simplest method is that of increasing the voltage used. The advantage of such a change was mentioned in the chapter on "Power Circuits," *i.e.*, the increase in load carried, with the same per cent voltage drop, will be equal to the square of the increase in voltage. A change in station transformers, station equipment and line transformers is necessary. The economical advantage can be studied by a careful consideration of the annual costs including, of course, the costs of making the change.

Another method is to change from a single-phase to a three-phase system for the main lighting circuits. The advantage gained is the well-known advantage of three-phase over single-phase. Twice the load can be carried for the same per cent power loss and per cent voltage drop. Balanced against this advantage is the cost of making station changes, of installing the third conductor, etc.

A combined increase in voltage and a change from single- to three-phase is sometimes used, such as a change, from a 2,300-volt, single-phase to a 4,600-volt, three-phase system. This method has the advantage of obtaining an increase in capacity, due to both changes with the cost of making the change very little greater than for either change alone.

A method which has been used recently in a number of places with apparent satisfaction is the installation of a four-wire, three-

phase system, with grounded neutral. For example, if 2,300-volt, single-phase circuits have been in use, the change is made to three-phase with a voltage of 2,300 volts from each phase to neutral. Some of the advantages claimed are:

1. The reduction in voltage drop and power loss due to three-phase transmission from substation to feeding points or branches.

2. The reduction in total number of conductors due to the combination of three former single-phase circuits with six conductors into one three-phase circuit with four conductors.

3. The utilization of the same branch single-phase circuits and distribution transformers without change.

4. The load need not be so well balanced as with three-phase, three-wire circuits on account of the use of a neutral conductor. Also the shutting down of one phase will not cut off service on the other phases.

5. The station changes are somewhat simpler than for a straight increase in voltage or change to a three-phase, three-wire system.

6. The ability to carry three-phase loads on the same circuit if desired.

Some disadvantages noted are the increase in voltage from phase wire to ground on single-phase branches, the concentration of fairly large loads on one circuit, and the fact that the increase in voltage is not as great as could be made by a straight doubling of voltage.

Another method which has also been tried out in a few places is that of extending the secondary transmission system. Instead of attempting to carry the load all out of main substations, on heavy feeder circuits, at primary voltage, small automatic or semi-automatic substations are established at desirable feeding points and these are supplied through high-voltage lines or cables. The primary circuits running from these substations are comparatively short and a corresponding advantage in regulation is accomplished.

No one of the above methods of increasing the capacity of the lighting circuit system can be recommended as most advantageous for all cases. The one best applicable to any system must be chosen by a careful study of present loads, conditions in the substations and on the lines, probable future loads, the cost and practical difficulties of making the change, and adaptability of the new system to still further changes when necessity de-

mands. This study should be based on as complete a determination as possible of annual costs before and after the change and the comparative annual costs with various alternative changes.

Importance of Good Service.—The importance of good service in connection with lighting circuits should be emphasized. It is here especially that a desire for economy should not lead the engineer to practices which will endanger the quality or continuity of the service rendered. A fluctuation of voltage is easily discernible in the effects on lighting and gives rise to many complaints. An interruption of service, especially if for any considerable length of time, discommodes a great number of customers, sometimes with very serious consequences. Economy is always desirable, but it is false economy to save a few dollars on construction and thereby lose customers. The aim should be to reduce cost to a minimum which is consistent with service, at least as good as that to which the customers have been accustomed.

CHAPTER XIII

SECONDARY DISTRIBUTION—SINGLE-PHASE

Study of Most Economical Design for Secondaries—Voltage Drop—Conductor Size—Transformer Size—Length of Secondary

In working toward the efficient and economical design of the central-station system as a whole no link in the chain connecting the consumer with the coal pile may be overlooked. The ultimate purpose of all study in this direction is to enable energy to be delivered to the customer at the least possible cost per unit, while at the same time good service is maintained. To this purpose considerable attention has been paid to generating plant, transmission lines and substations but on the final link before reaching the customer—the distribution lines—the tendency has been to apply "rule of thumb" methods and "experience" only to the layouts. When it is considered that, even in a well-designed system, the investment in distribution lines will often be from one-fifth to one-fourth of the total investment on the system, and that the energy losses on these lines will be equal to or somewhat more than one-half of the total loss between the generator and the customer, it may be expected that a study of the economical design of distribution lines will be found of great profit.

There are several conditions pertaining to the secondary system which make the careful layout of such a system especially important. The number of transformer installations is so large and the lines spread over so great an area that constant or very frequent inspection is impossible. The load is subject to irregular increases. In districts which are newly built up, new services are constantly being added. In old districts, new appliances are being purchased and the load on old services thereby increased. On this account any design must be made to cover a period of years and the increase in load for that period estimated from past experience. On the other hand, care must be taken not to install too much capacity and thereby increase the cost beyond the limits of economy. The problem must be carefully studied to obtain the balance between low cost and good service for any particular case.

The problems of secondary distribution economics deal chiefly with the wire size and the size and arrangement of transformers. The voltage is usually limited to a small range of values by past practice, transformer standards, lamps, motor and other appliance standards, etc. The layout of pole locations, while requiring a considerable amount of engineering skill and experience, is usually dependent on local conditions, the arrangement of lot lines, convenience in reaching services, probable future business, etc. rather than on purely economic considerations. Of course, many local problems arise in the layout and construction of secondary lines, in the solution of which economics should be considered. Changes in type of construction should be looked at from an economic view point as well as from that of the mechanical design only.

There are several general types of problems relating to secondaries. They might be classified in general as those of:

(*a*) Single-phase secondaries in cities and large towns.
(*b*) Single-phase secondaries in small towns and country.
(*c*) Three-phase secondaries on large power installations.
(*d*) Three-phase secondaries on small power installations.

In this chapter the problem of single-phase secondaries will be discussed, especially in reference to well built up districts where the load may be considered to have practically a uniform distribution.

Secondaries for Uniformly Distributed Load.—In attacking such a problem we can often determine from tests and from past experience what the density of the loading is and how it will increase for some years in advance. We are usually limited to certain stock sizes of transformer and of wire, on any system, due to practical considerations of manufacturing and stock keeping. The problem then is to determine the proper combination of wire, transformer and transformer spacing in order to give good conditions of operation and also to show the least cost per year for the load densities expected during the period of time under consideration. It is clearly understood that no definite rules can be established which will fit all conditions. The variations in the problem are too many. The most that can be done is to furnish means for readily discovering the limitations of any problem and of proceeding within these limitations to the most economical design.

The study has been carried forward from three different angles. First, from the theoretical; second, from a semi-practical, that is, by adopting certain standards and studying their behavior; third, from a purely practical, giving the designer data on the costs of various transformers and wire sizes under the conditions ordinarily encountered in practice.

In all this discussion it has been assumed that the loading is such that it may be considered as uniformly distributed along the line. The unit used is called load density, given in kilowatts per 1,000 ft. The line is assumed to be three-wire secondary spaced 42 in. between outside wires. The cost of right-of-way, poles, crossarms and insulators is not included in any of the computations as it is assumed this would be the same under any given condition. Also the difference in length of primary for different transformer spacings is not considered. In actual design under known conditions a correction should be made for this. The loading conditions are taken as those of residence-lighting districts although the same methods could be adapted to any other conditions of loading if its characteristics were known. Transformers are assumed to be in the center of the secondary served, feeding both ways.

DISCUSSION OF METHODS USED IN DERIVING EQUATIONS AND THEIR APPLICATION

Theoretical.—In the theoretical discussion ideal conditions are assumed which will rarely if ever be met with in practice, but it can be shown by a study of the results how they may be applied to practical conditions. These assumptions are that the line is indefinite in length so that the transformers may be placed at any exactly determined spacing and that the spacing will change with the load; that the transformer is always of a size just equal to the load to be carried, that is, equal to the load density at peak load times the spacing; that the wire may be of any cross sectional area and vary with the load. Such a condition could only be obtained in a case where the load showed only seasonal variations and no yearly increase. However, in practice we usually design for a certain period at the end of which it is assumed the load density will be a certain amount.

The general method has been to obtain an expression for the annual cost per 1,000 ft. of line and to determine by finding the

first derivative and setting it equal to zero, the condition under which this annual cost is a minimum. This is the most economical condition.

Annual Cost of Secondary Distribution.—The general equation for the annual cost is first obtained as follows:

Annual cost per 1,000 ft. of installation $= Y$

$Y =$ (Total annual charges on transformers per 1,000 ft. of line) + (Total annual charges on line per 1,000 ft. of secondary) $= Y_T + Y_L$ (55)

Where $Y_T = \left(\frac{g_T}{100}\right.$ (Purchase price + cost of handling + cost of installation + cost of lightning arresters and equipment) + Cost of core and copper loss + cost of inspection$\left.\right)\frac{1000}{S}$

$g_T =$ Per cent interest + depreciation + taxes on transformer.

$S =$ Spacing of transformers in feet.

The core loss is practically a constant quantity for 24 hr. per day throughout the year. The copper loss on the other hand depends on the load. If the characteristic variation of this load from hour to hour, day to day and month to month is known, the average loss per day can be determined in terms of the year's peak load. In this case the peak load is assumed to be just equal to the capacity of the transformer. The cost of energy at the transformer must also be carefully determined. The cost for copper loss will be considerably higher than that for core loss on account of the lower load factor. The sum of all these items makes up the annual cost on a transformer.

It was found that if the value of the transformer annual cost is plotted against the transformer size that the curve for values between 0 and 25 kw. may be approximated by a straight line of the formula $Y_T' = K_1 + K_2T$, T being the transformer size and K_1 and K_2 constants to be determined for any particular combination of transformer cost, energy cost, etc. (see Fig. 36). This becomes $Y_T = \frac{1,000}{S}(K_1 + K_2T)$ per 1,000 ft., where S is the length of secondary belonging to any one transformer or the distance between transformers where banked.

Assuming a transformer size just sufficient to carry the load, then

$$T = L_D \frac{S}{1{,}000}$$

Where L_D = load density in kw. per 1,000 ft.

$$\text{Then } Y_T = \frac{1{,}000}{S}\left(K_1 + K_2 \frac{L_D S}{1{,}000}\right) \tag{56}$$

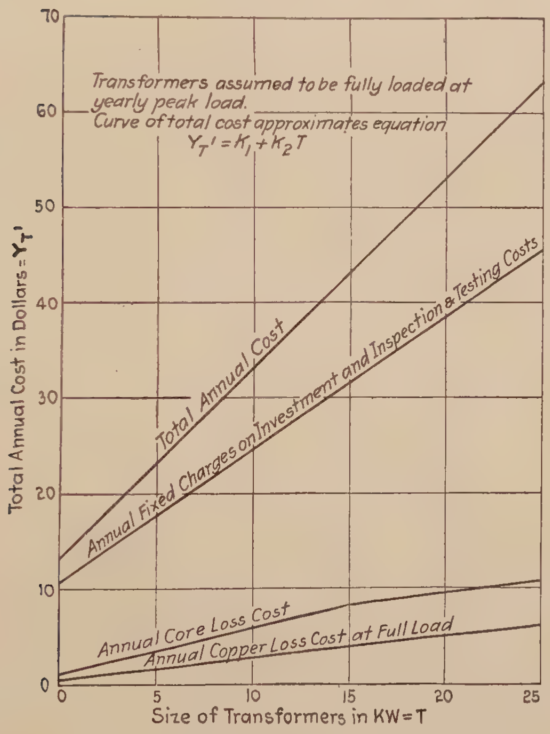

FIG. 36.—Annual charges on transformers.

The annual cost on the line includes interest, depreciation and taxes on the investment cost of the wire in place, including purchase price and cost of installation, also the cost of annual energy loss due to resistance. The copper loss is arrived at by the same method as the copper loss on the transformer, that is, by use of the equivalent average number of hours per day at full load or equivalent hours.

Y_L = Investment cost of material per 1,000 ft. of line + installation charges per 1,000 ft. of line + $\frac{1,000}{S}$ × cost of copper loss in secondary.

$$Y_L = \frac{g_L}{100}(3 \times 1{,}000 \times w \times C_{cu} + C_{sr}) + \frac{1{,}000}{S}\left[2\left(\frac{I}{2}\right)^2 \times \frac{\rho}{A} \times \frac{S}{6} \times 2 \times t \times 365 \times \frac{C_{e2}}{1{,}000}\right].$$

Where g_L = per cent interest + depreciation + taxes on line,
w = weight of insulated wire in pounds per foot,
C_{cu} = cost of insulated wire per pound,
C_{sr} = cost of stringing 1,000 ft. of line,
I = total current in secondary at transformer,
ρ = resistivity of wire per mil foot,
C_{e2} = cost of copper loss in secondary per kilowatt-hour,
t = equivalent hours per day which yearly peak load should continue in order to give an I^2R loss equal to the total actual I^2R loss for the year,
A = cross-sectional area of wire in circular mils,
E = voltage between outside wires of secondary,
Cos θ = power factor of load.

$$I = \frac{L_D S}{E \cos \theta}$$

and

$$Y_L = \frac{g_L}{100}(3{,}000\, wC_{cu} + C_{sr}) + S^2\left[\frac{L^2_D}{E^2 \cos^2\theta} \times \frac{\rho}{A} \times t \times 60.83 C_{e2}\right] \quad (57)$$

The total annual cost per 1,000 ft. of installation is now obtained by adding these two quantities, annual cost of transformers and annual cost on line, and the equation obtained as shown below.

$$\text{Then } Y = \frac{1{,}000}{S}\left(K_1 + \frac{K_2 L_D S}{1{,}000}\right) + \frac{g_L}{100}(3{,}000\, wC_{cu} + C_{sr}) + S^2\left(60.83\, \frac{L_D^2 \rho t C_{e2}}{AE^2 \cos \theta}\right) \quad (58)$$

Equation 58 gives the total annual cost per 1,000 ft. of installation as a function of the spacing and load density.

Most Economical Voltage Drop.—One of the most important controlling factors in determining the length of a secondary or the spacing of transformers is the maximum allowable voltage drop.

It has usually been considered that the most economical condition of operation would be with a voltage drop higher than would be allowable for good service. In our case, 3 per cent drop has been considered the limiting value, as luminosity curves for Mazda lamps show a reduction as high as 18 per cent with 5 per cent voltage drop while 3 per cent shows over 10 per cent reduction. Considering the voltage loss in the service drops, which cannot be figured closely on account of variable conditions and the fact that the load is not absolutely uniformly distributed, 3 per cent is considered the highest value commensurate with good operation. It must be determined, then, if under certain conditions, a smaller voltage drop than this will be more economical.

In order to obtain the most economical per cent voltage drop it is necessary to obtain Y as a function of the per cent voltage drop.

This is done as follows:

W = total load on secondary in watts,
V = voltage drop on secondary in per cent of delivered voltage,
B = constant relation between per cent voltage drop and the per cent power loss.

$$\text{Then } W = L_D S = \frac{AE^2V}{300\ BS}$$

$$\text{Whence } S = \frac{E}{17.32}\sqrt{\frac{AV}{BL_D}}$$

Substituting this value for S in Eq. 58,

$$\text{Then } Y = \frac{1{,}000\ K_1}{\dfrac{E}{17.32}\sqrt{\dfrac{AV}{BL_D}}} + K_2L_D + \frac{g_L}{100}(3{,}000\,w\,C_{cu} + C_{sr}) + \frac{E^2}{(17.32)^2}\,\frac{AV}{BL_D}\left[60.83\,\frac{L_D{}^2\rho t C_{e2}}{AE^2\cos^2\theta}\right]$$

$$= 17{,}320\,\frac{K_1}{E}\sqrt{\frac{BL_D}{A}}\,V^{-\frac{1}{2}} + 0.2028\,\frac{\rho t C_{e2} L_D}{B\cos^2\theta}\,V + K_2L_D + \frac{g_L}{100}(3{,}000 \times C_{cu} + C_{sr}) \quad (59)$$

The most economical per cent voltage drop is obtained when the first derivative of Y with respect to V equals 0

$$\frac{dY}{dV} = 0$$

$$= -\tfrac{1}{2} = 17{,}320\ K_1\sqrt{\frac{BL_D}{A}}\,V_{ec}{}^{-\frac{3}{2}} + 0.2028\,\frac{\rho t C_{e2} L_D}{B\cos^2\theta} = 0$$

Solving for V_{ec}

$$V_{ec} = 1{,}223 \left[\frac{\cos^2 \theta}{E\rho t C_{e2}}\right]^{2/3} \frac{BK_1^{2/3}}{A^{1/3}L_D^{1/3}} \tag{60}$$

Equation 60 gives the most economical per cent voltage drop as a function of the load density.

By assuming values for the constants to fit particular conditions this expression for V can be plotted against load density for various standard wire sizes. These curves show that, as load density increases, the most economical voltage drop decreases and, under the conditions assumed in the curves here plotted, the most economical voltage drop falls below 3 per cent at load densities which are often encountered with such loads (see Fig. 37).

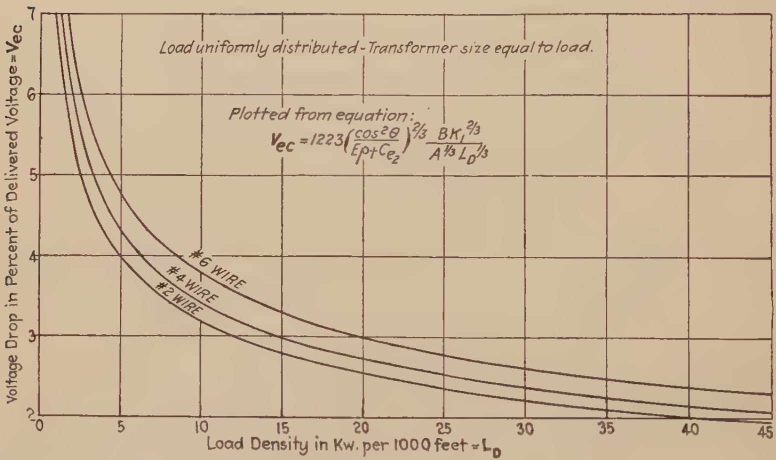

FIG. 37.—Most economical voltage drop in per cent of delivered voltage.

Most Economical Transformer Spacing.—In order to obtain the most economical length of secondary or spacing of transformers it is necessary to have Y as a function of S. This is obtained from Eq. 58.

$$Y = \frac{1{,}000K_1}{S} + K_2L_D + \frac{g_L}{100}(3{,}000\, wC_{cu} + C_{sr}) + S^2\,\frac{(60.83\, L_D^2 \rho t C_{e2})}{AE^2 \cos^2\theta}$$

The most economical spacing is obtained when the first derivative of V with respect to S equals O —

$$\frac{dY}{dS} = 0$$

$$= -\frac{1{,}000\,K_1}{S_{ec}^{\,2}} + 2 \times 60.83\frac{L_D{}^2\rho t C_{e2}}{AE^2\cos^2\theta}S_{ec} = 0$$

Solving for S_{ec}

$$S_{ec} = 2.02\left[\frac{K_1E^2\cos^2\theta}{\rho t\,C_{e2}}\right]^{\frac{1}{3}}\frac{A^{\frac{1}{3}}}{L_D{}^{\frac{2}{3}}} \qquad (61)$$

Equation 61 gives the most economical spacing of transformers as a function of the load density.

It is necessary to limit the range of application of Eq. 61 to conditions where the voltage drop is less than 3 per cent. A second equation must be developed for 3 per cent drop to apply where the most economical drop would be greater than 3 per cent. Practical considerations limit the drop to that value. From above

$$S = E\sqrt{\frac{AV}{300\,BL_D}}$$

Using $V = 3$ per cent

$$S = \frac{E}{10}\sqrt{\frac{A}{BL_D}} \qquad (62)$$

= Transformer spacing for 3 per cent drop.

Then, summarizing,

$$S_{ec} = 2.02\left[\frac{K_1E^2\cos^2\theta}{\rho t\,C_{e2}}\right]^{\frac{1}{3}}\frac{A^{\frac{1}{3}}}{L_D{}^{\frac{2}{3}}} < \frac{E}{10}\sqrt{\frac{A}{BL_D}} \qquad (63)$$

which is general for all cases. If now the constants are evaluated these curves may be plotted for various sizes of wire, using, for any particular load density, the equation which shows the shortest spacing. We obtain the set of curves, Fig. 38, giving the transformer spacing which will give, with any wire size, the greatest economy, providing good operating conditions are maintained by having no voltage drop greater than 3 per cent.

Most Economical Transformer Size.—It is a simple matter with this data at hand to derive the curves showing the most economical transformer size for any load density, providing the transformers are spaced most economically. Since it was assumed in the beginning that the transformer would be just large enough to carry the load, $T_{ec} = L_D\frac{S_{ec}}{1{,}000}$ (Eq. 64) where S_{ec}

is the value taken from the curves for most economical spacing. This is the most economical size for any load since the annual charges on the investment represented by the transformer is a

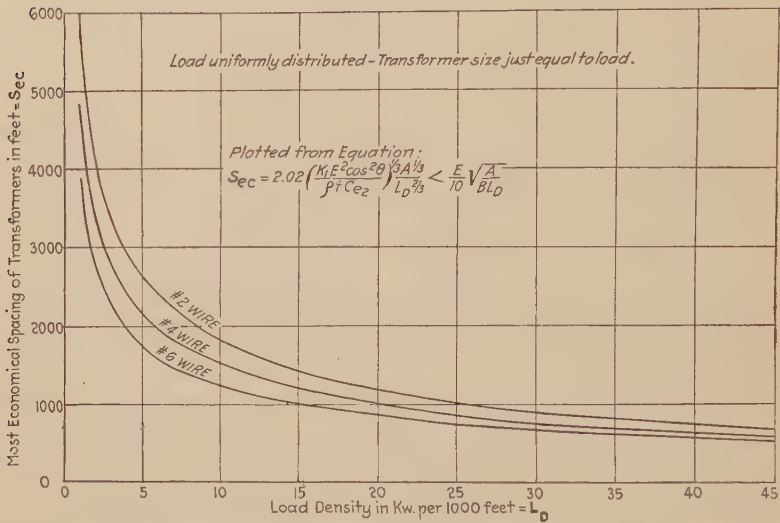

Fig. 38.—Most economical spacing of transformers (limited by a maximum allowable voltage drop of 3 per cent).

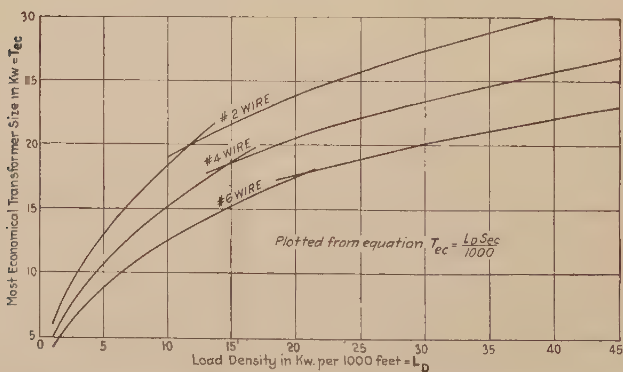

Fig. 39.—Most economical transformer size (being just equal to the load at the most economical spacing).

much greater proportion of the total annual charge than the cost of energy losses. Therefore the use of a larger transformer, even though under-loaded, would be more costly (see Fig. 39).

Most Economical Wire Size.—It is now possible to attack the problem of the most economical size of wire for any load density. We will assume that it is feasible to use the most economical transformer size at its most economical spacing for any load density, modified by the limiting 3 per cent voltage drop requirement. Then if we substitute in our original equation (Eq. 58) the expressions for S used in plotting the curves for most economical spacing and for spacing limited by 3 per cent drop in voltage, we obtain two expressions for the annual cost per 1,000 ft. in terms of load density and cross-sectional area of wire (A). It is necessary to introduce two approximations. The weight per foot of wire (w) enters the equation, also the quantity B which

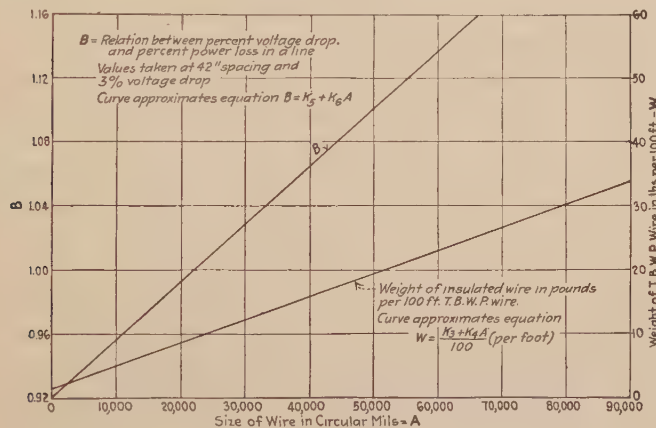

Fig. 40.

is the constant relation between per cent voltage drop and per cent power loss for any size of wire. It is found by plotting values of w for standard sizes of wire of the range of sizes which would be used in secondaries that the expression $W = K_3 + K_4A$ is a very close approximation, K_3 and K_4 being constants (see Fig. 40). Also it is found that the value of B for any size of wire may be approximated very closely by the straight line function $B = K_5 + K_6A$, where K_5 and K_6 are constants (see Fig. 40). These must be derived from the particular values of B which apply to the conditions being studied since these values vary for different spacings between wires. Substituting these expressions in the equations referred to above we obtain the two general

expressions for annual cost per 1,000 ft. in terms of wire size for maximum economy of transformer spacing and for 3 per cent voltage drop, Eqs. 65 and 67. These are differentiated with respect to A and the Eqs. 66 and 68 are obtained between the most economical wire size and the load density for most economical spacing and for 3 per cent voltage drop.

Substituting the value S_{ec} (Eq. 61) for S in Eq. 58

$$Y = \frac{1{,}000\,K_1}{2.02\left(\dfrac{K_1E^2\cos^2\theta}{\rho tC_{e2}}\right)^{1/3}\dfrac{A^{1/3}}{L_D^{2/3}}} + K_2L_D + \frac{g_L}{100}(3{,}000wC_{cu} + C_{sr})$$
$$+ 60.83\,\frac{L_D{}^2\rho tC_{e2}}{AE^2\cos^2\theta}\left[2.02^2\left(\frac{K_1E^2\cos^2\theta}{\rho tC_{e2}}\right)^{2/3}\frac{A^{2/3}}{L_D^{4/3}}\right]$$

If $w = \dfrac{K_3 + K_4A}{100}$ (see curve 40).

The equation for annual costs per 1,000 ft. becomes

$$Y = \frac{496(K_1{}^2\rho tC_{e2})^{1/3}L_D^{2/3}}{(E^2\cos^2\theta)^{1/3}}A^{-1/3} + K_2L_D + \frac{g_L}{100}\Big[30(K_3+K_4A)C_{cu}+C_{sr}\Big]$$
$$+ 248.5\left(\frac{K_1{}^2\rho tC_{e2}}{E^2\cos^2\theta}\right)^{1/3}L_D^{2/3}A^{-1/3}$$

Simplifying

$$Y = 744.5\left(\frac{K_1{}^2\rho tC_{e2}}{E^2\cos^2\theta}\right)^{1/3}L_D^{2/3}A^{-1/3} + K_2L_D$$
$$+ \frac{g_L}{100}\Big[30(K_3 + K_4A)C_{cu} + C_{sr}\Big] \qquad (65)$$

Equation 65 gives the annual cost per 1,000 ft. of line, using the most economical spacing of transformers.

The most economical cross-section of wire is obtained when the first derivative of Y with respect to A equals O or

$$\frac{dY}{dA} = 0$$
$$= -\frac{1}{3}\times 744.5\left[\frac{K_1{}^2\rho tC_{e2}}{E^2\cos^2\theta}\right]^{1/3}L_D^{2/3}A_{ec}^{-4/3} + \frac{g_L}{100}\times 30K_4C_{cu}$$

Solving for A_{ec}

$$A_{ec} = \frac{154}{g_LK_4C_{cu}^{3/4}}\left[\frac{K_1{}^2\rho tC_{e2}}{E^2\cos^2\theta}\right]^{1/4}L_D^{1/2} \qquad (66)$$

Equation 66 gives the most economical cross-section of wire using the most economical spacing of transformers.

It is necessary to limit the application of Eq. 66 to less than 3 per cent voltage drop and develop the equation for most economical wire size with 3 per cent drop. This is done as follows:

From Eq. 62, the spacing which will give a 3 per cent voltage drop is,

$$S = \frac{E}{10}\sqrt{\frac{A}{BL_D}}$$

B may be expressed as a function of A as follows:

$$B = K_5 + K_6A \text{ (see curve 40).}$$

$$S = \frac{E}{10}\sqrt{\frac{A}{(K_5 + K_6A)L_D}}$$

Substituting the value of S in Eq. 58, the expression for annual costs per 1,000 ft. of line (the spacing being limited for a 3 per cent voltage drop) becomes

$$Y_{3\text{ per cent}} = \frac{1{,}000K_1}{\frac{E}{10}\sqrt{\frac{A}{(K_5 + K_6A)L_D}}} + K_2L_D + \frac{g_L}{100}[30(K_3 + K_4A)C_{cu} + C_{sr}] + 60.83\,\frac{L_D{}^2\rho tC_{e2}}{AE^2\cos^2\theta} \times \frac{E^2}{100}\,\frac{A}{(K_5 + K_6A)L_D}$$

Simplifying

$$Y_{3\text{ per cent}} = \frac{10{,}000K_1L_D{}^{1/2}}{E}\left(\frac{K_5}{A} + K_6\right)^{1/2} + K_2L_D + \frac{g_L}{100}[30(K_3 + K_4A)C_{cu} + C_{sr}] + 0.6083\,\frac{\rho tC_{e2}L_D}{\cos^2\theta(K_5 + K_6A)} \tag{67}$$

Equation 67 gives annual cost per 1,000 ft. of line using a spacing which limits the voltage drop to 3 per cent at full load.

The most economical cross-sectional area is obtained when the first derivative of $Y_{3\text{ per cent}}$ with respect to A is equal to 0.

$$\frac{dY_{3\text{ per cent}}}{dA} = 0$$

$$= \frac{1}{2} \times \frac{10{,}000K_1L_D{}^{1/2}}{E}\left(\frac{K_5}{A_{ec}} + K_6\right)^{-1/2}\left(-\frac{K_5}{A^2_{ec}}\right) + \frac{g_L}{100} \times 30K_4C_{cu} - .6083\,\frac{\rho tC_{e2}L_D}{\cos^2\theta(K_5 + K_6A_{ec})^2}K_6$$

From which

$$\frac{5{,}000K_1K_5L_D{}^{1/2}}{E\left(\frac{K_5}{A_{ec}} + K_6\right)^{1/2} A^2_{ec}} + .6083\frac{K_6\rho tC_{e2}L_D}{\cos^2\theta(K_5 + K_6A_{ec})^2} = .30g_LK_4C_{cu} \tag{68}$$

Equation 68 gives the most economical cross-sectional area of wire when the spacing is limited by a 3 per cent voltage drop.

The constants were evaluated and these curves plotted, the 3 per cent curve for low load densities and the maximum economy curve for high loading. They furnish a graphic representation of the most economical size of wire to use under any load density providing ideal conditions obtain in the way of transformer size and spacing. See Fig. 41.

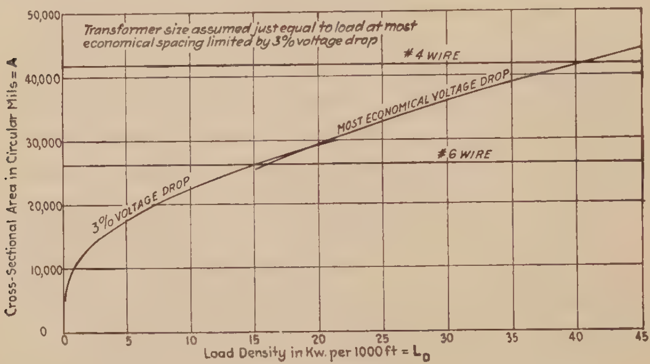

FIG. 41.—Most economical wire size.

Purpose of Theoretical Curves.—At first glance it may appear as if these curves, being obtained on the basis of such theoretical assumptions, could have very little practical value. However, when attacking a practical problem of this nature the data from these curves may be used as the basis upon which to start the calculations of annual costs under operating conditions. If, for example, the present load density and the load density which is to be expected at some certain future time are known, by going to the theoretical curves we may determine (*a*) whether the voltage drop is to be limited by the 3 per cent maximum, (*b*) what would be the most economical conditions of transformer size and spacing for present operation and for operation at that future time, and (*c*) what standard size of wire will be most economical over the period. The curve for the most economical wire size covers, for each standard size, such a range of load densities that we should be able at once to select our wire size without further computation. Having determined this and knowing what stock sizes of transformers and practical spacings come the nearest to fitting the ideal conditions over the period under

consideration, we can then investigate, as will be shown later, the comparative economy of such various methods of installation as could be used in this particular case. In other words,

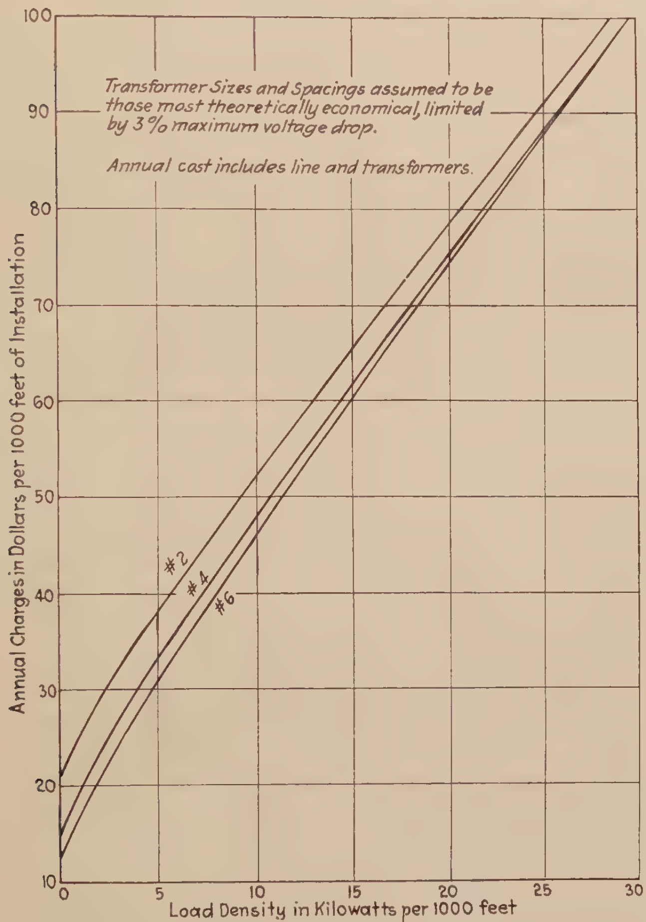

FIG. 42.—Curves showing comparative economy of various wire sizes in secondary installations.

these theoretical curves give certain limitations on which we may proceed to further more practical investigation.

Semi-practical.—In order to present our results in a little more concrete and practical form and to show the exact comparative economy between various types of installation, espe-

cially with respect to the size of wire to be used, a series of curves were developed showing the exact annual cost under various conditions. These are called the semi-practical curves (Figs. 42 and 43).

Annual Cost of Standard Wire Sizes Working under Ideal Conditions.—The first condition was assumed to be that in which the most economical size of transformer could be used, spaced the most economically or, where necessary, for 3 per cent maximum voltage drop. A curve was plotted for each of the three standard sizes of wire, No. 6, No. 4 and No. 2, showing the annual cost at various load densities (see Fig. 42). This is, in reality, simply plotting Eqs. 65 and 67 as developed above.

Annual Costs per 1,000 ft. of Installation for Any Combination of Standard Sizes of Wire and Transformers.—As the next step in proceeding from the general problem to the concrete example various combinations of standard sizes of transformers with standard sizes of wire were assumed and curves developed showing the annual cost of each of these combinations at various load densities. The transformer spacing was still assumed to be always the theoretically best spacing for each particular load. This enables us to compare for example the economy of a 10-kw. transformer and No. 4 wire with that of a 15-kw. and No. 6 wire at any load density.

The method of developing these curves has some points of interest although the equations are merely variations of our general equation for annual cost per 1,000 ft. It is seen that for any size of transformer, as the load density increases a certain point is reached where the spacing is no longer governed by the allowable voltage drop but by the size of the transformer itself. Hence each curve will consist of two parts, the lower where the voltage drop governs the spacing, and excess transformer capacity is provided, the upper where the transformer size governs the spacing and the voltage drop is less than the allowable. The total annual cost is made up of five items:

1. Transformer core loss.
2. Transformer copper loss.
3. Copper loss on the line itself.
4. Fixed charges on the transformer (interest, depreciation, taxes, inspecting, tests, etc.).
5. Fixed charges on the line. (Interest and depreciation.)

Each of these five elements was analyzed as to constants and

variables, considering the load density L_D as the chief variable, and the transformer and wire sizes constant for any given condition. It was found that the equations took the following form:

$$Y = K_8 L_D^{3/2} + K_9 L_D + (K_7 + K_{10}) L_D^{1/2} + K_{11}$$

when the voltage drop and wire size governs, and

$$Y = (K_{12} + K_{13} + K_{15}) L_D + (K_{11} + K_{14})$$

when transformer size governs.

The first is an equation of a third degree curve in $L_D^{1/2}$ breaking into a straight line (the second equation) at the critical point where the spacing for 3 per cent drop fully loads the transformer. The equation for each constant was then developed and evaluated for each combination of wire and transformer. The expressions for costs here given differ from those given in the theoretical discussion in that here actual stock sizes of transformers and standard wire sizes are used. The results were then plotted as shown in Fig. 43. The derivation of these curves is a good example of the method of developing a general curve by the use of symbols for all constants and then evaluating these symbols to fit a given condition.

The derivation follows:

The load possible on a given wire with a 3 per cent drop is given by the formula:

$$W = \frac{3AE^2}{B \times 300 \times S} = \frac{AE^2}{100BS}$$

$$W = L_D S \text{ and } I = \frac{L_D S}{E \cos \theta}$$

$$S = \frac{E}{10}\sqrt{\frac{A}{BL_D}} \text{ when limited by voltage drop (see Eq. 62).}$$

$$\text{The total load on the transformer } = \frac{L_D S}{1{,}000}$$

$$S = \frac{1{,}000T}{L_D} \text{ when limited by transformer capacity.}$$

The following items enter into the total annual charges per 1,000 ft. of installation.

(*a*) *Transformer Core Loss.*—This is assumed constant for a given transformer for all loads.

Core loss = constant.

$$\text{Annual charge per 1,000 ft.} = \frac{1{,}000}{S} \times C_{e1} \times 24 \times 365 \times \frac{\text{core loss}}{1{,}000}$$

$$= \frac{C_{e1} \times 24 \times 365 \times 10 \times \text{core loss}}{E\sqrt{\frac{A}{B}}} \times L_D^{1/2} \text{ (limited by voltage drop)}$$

$$= K_7 L_D^{1/2}$$

or

$$= \frac{C_{e1} \times 24 \times 365 \times \text{core loss}}{1{,}000T} L_D \text{ (limited by transformer size)}$$

$$= K_{12} L_D$$

Whence,

$$K_7 = \frac{C_{e1} \times 24 \times 365 \times 10 \times \text{core loss}}{E\sqrt{\frac{A}{B}}}$$

$$K_{12} = \frac{C_{e1} \times 24 \times 365 \times \text{core loss}}{1{,}000T}$$

C_{e1} = cost of core loss per kilowatt-hour

Core loss in watts.

(b) *Transformer Copper Loss.*—

$$= \frac{1{,}000}{S}\left(I^2R_T \times C_{e2} \times \frac{365 \times t}{1{,}000}\right)$$

$$= \frac{1{,}000}{S}\left(\frac{L_D{}^2S^2}{E^2\cos^2\theta}R_T \times C_{e2} \times \frac{365 \times t}{1{,}000}\right) = \frac{L_D{}^2R_TC_{e2} \times 365tS}{E^2\cos^2\theta}$$

$$= \frac{R_TC_{e2} \times 365 \times t\sqrt{\frac{A}{B}}}{10E\cos^2\theta} L_D^{3/2} \text{ (limited by voltage drop)}$$

$$= K_8 L_D^{3/2}$$

$$= \frac{R_TC_{e2} \times 365 \times 1{,}000 \times t \times T}{E^2\cos^2\theta} L_D \text{ (limited by transformer size)}$$

$$= K_{13} L_D$$

Where

$$K_8 = \frac{R_TC_{e2}\ 365t\sqrt{\frac{A}{B}}}{10E\cos^2\theta}$$

$$K_{13} = \frac{R_TC_{e2}\ 365{,}000t \times T}{E^2\cos^2\theta}$$

(c) *Secondary Copper Loss.*—

$$R = \rho\frac{S}{A}$$

$$\text{Secondary copper loss} = \frac{1{,}000}{S} 2 \left(\frac{I}{2}\right)^2 \times \frac{\rho}{A} \times \frac{S}{6} \times 2 \times t \times 365 \times \frac{C_{e2}}{1{,}000}$$

$$= \frac{L_D{}^2}{E^2 \cos^2 \theta} \frac{\rho t}{A6} 365 C_{e2} S^2$$

$$= \frac{\rho t 365 C_{e2}}{\cos^2 \theta B 600} L_D \text{ (when limited by voltage drop)}$$

$$= K_9 L_D$$

or

$$= \frac{\rho t 365 C_{e2} 10^6 T^2}{E^2 \cos^2 \theta A} \text{ (when limited by transformer size)}$$

$$= K_{14}$$

Where

$$K_9 = \frac{\rho t 365 C_{e2}}{\cos^2 \theta B 600}$$

$$K_{14} = \frac{\rho t 365 C_{e2} 10^6 T^2}{E^2 \cos^2 \theta A}$$

(*d*) *Fixed Charges on Transformer.*—Constant for any size of transformer.

$$= \frac{1{,}000}{S} \left[\frac{g_T}{100} \text{(Transformer cost + lightning arrester + cost of installation) + inspection cost} \right]$$

$$= \frac{10^4}{E\sqrt{\frac{A}{B}}} \left[\frac{g_T}{100} \text{(Transformer cost + lightning arrester + cost of installation) + inspection cost} \right] L_D{}^{1/2}$$

(when limited by voltage drop)

$$= K_{10} L_D{}^{1/2}$$

or

$$= \frac{1}{T} \left[\frac{g_T}{100} \text{(Transformer cost + lightning arrester + cost of installation) + inspection cost} \right] L_D$$

(when limited by transformer size)

$$= K_{15} L_D$$

Where

$$K_{10} = \frac{10^4}{E\sqrt{\frac{A}{B}}} \left[\frac{g_T}{100} \text{(Transformer cost + lightning arrester + cost of installation) + inspection cost} \right]$$

$$K_{15} = \frac{1}{T} \left[\frac{g_T}{100} \text{(Transformer cost + lightning arrester + cost of installation) + inspection cost} \right]$$

(*e*) *Fixed Charges on Wire.*

$$= \frac{g_L}{100} (3{,}000 w C_{cu} + C_{sr})$$

$$= K_{11}$$

(*f*) *Total Annual Cost.*—The total yearly cost per 1,000 ft. of line is the summation of these five items.

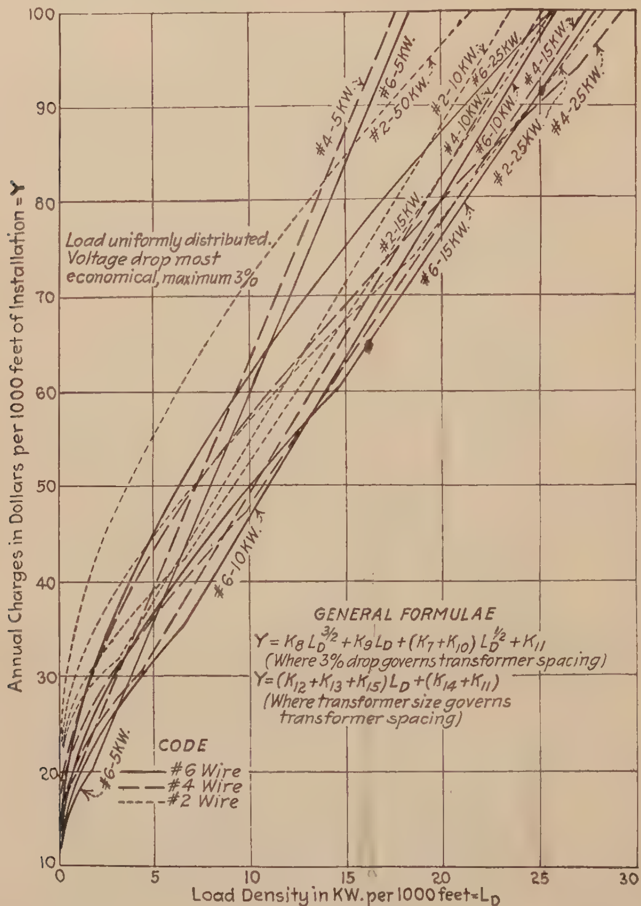

FIG. 43.—Curves showing comparative economy of various combinations of secondary installations.

Hence

$$Y_3 \text{ per cent} = K_7L_D^{1/2} + K_8L_D^{3/2} + K_9L_D + K_{10}L_D^{1/2} + K_{11}$$
$$= K_8L_D^{3/2} + K_9L_D + (K_7 + K_{10})L_D^{1/2} + K_{11} \quad (69)$$

(when limited by the voltage drop)

$$Y = K_{12}L_D + K_{13}L_D + K_{14} + K_{15}L_D + K_{11}$$
$$= (K_{12} + K_{13} + K_{15})L_D + (K_{14} + K_{11}) \quad \text{(when limited by transformer size)} \quad (70)$$

(see curve Fig. 43).

Purpose of Semi-practical Curves.—These semi-practical curves, although reducing the variable elements, still retain enough of the ideal condition so that they cannot be used as an absolute criterion but merely as a guide. They do show however concretely the relative economy of the various standard sizes of wire when used under the most favorable conditions and this may be taken as a guide to their comparative behavior under all conditions. The second set of curves also shows concretely the relative economy of the various transformer sizes with any one

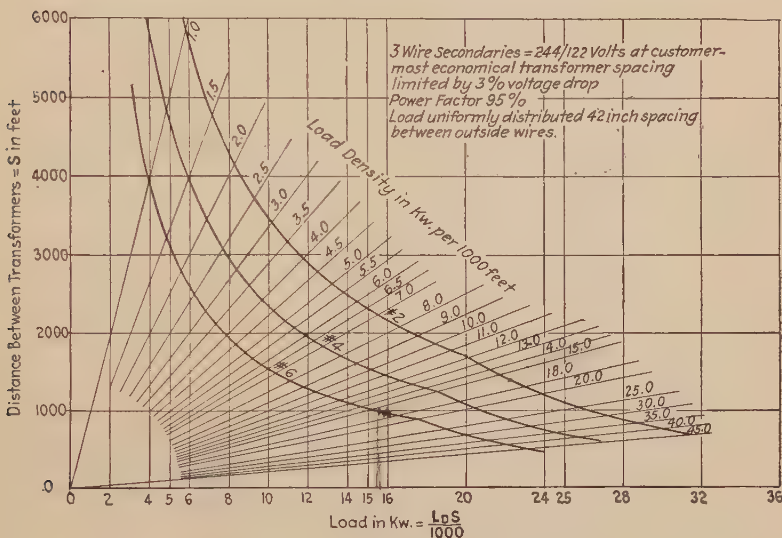

FIG. 44.—Load curves for secondaries.

size of wire as well as the relative economy of various sizes of wire with any size of transformer. This comparison of economy is valuable in showing the exact amount which the annual cost of one installation is greater or less than another. It often occurs that where the difference in cost is not great, other advantages are sufficient to more than offset it and lead to the choice of the more costly. The spacing of transformers is here considered to be the maximum allowable throughout, with the transformer carrying its maximum allowable load. This limits the general application of these curves in practice and hence like the first series they are chiefly useful in establishing limits and as a basis for the design.

Practical.—We now come to the development of the curves which the designer may use in testing the economy of any design and thereby choose the most economical from several alternatives. Here no "most economical" conditions need be assumed. The curves simply represent annual costs as they occur under any condition which may be encountered.

Load Curves for Secondaries.—The first curve is a development from the two theoretical curves, the most economical transformer spacing and most economical transformer size.

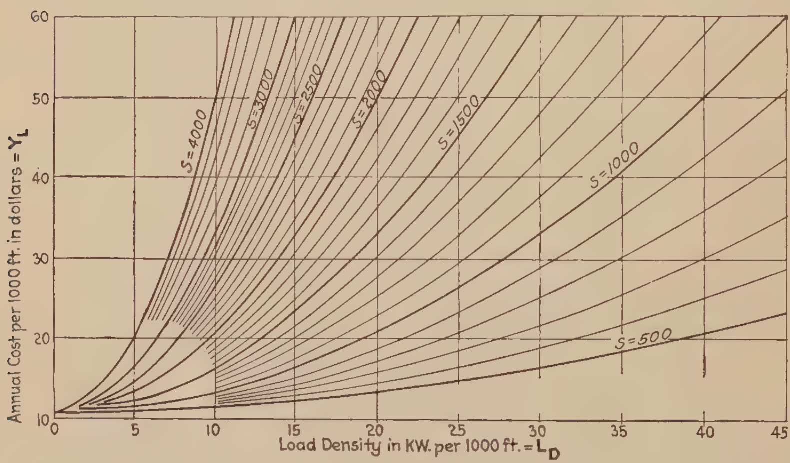

FIG. 45*a*.—Line cost curves. Annual cost per 1,000 ft. of 3 No. 6 secondaries including fixed charges and cost of lost energy.

From Eq. 63

$$S_{ec} = 2.02\left(\frac{K_1E^2\cos^2\theta}{ptC_{e2}}\right)^{1/3}\frac{A^{1/3}}{L_D^{1/3}} < \frac{E}{10}\sqrt{\frac{A}{BL_D}}$$

By plotting the transformer size against the spacing we obtain for each size of wire a curve showing the most economical spacing or the spacing limited by 3 per cent voltage drop for any total load on the transformer (see Fig. 44). By drawing diagonal lines, one for each load density desired, we can now show for any particular load density, the maximum economical spacing, and the minimum transformer size with that density and spacing. This curve merely simplifies the former two and serves the same purpose, not introducing any new principles. It is evident that any point below the curve will indicate a drop less than the value used on the curve. This curve is of use in determining what

alternative designs may be feasible with any load and standard equipment and what changes may be made to care for an increase.

Line Cost Curves.—The equation for annual charges on the line (Eq. 57) is next developed numerically.

$$Y_L = \frac{g_L}{100}(3{,}000wC_{cu} + C_{sr}) + 60.83\frac{L_D{}^2S^2\rho tC_{e2}}{AE^2\cos^2\theta}$$

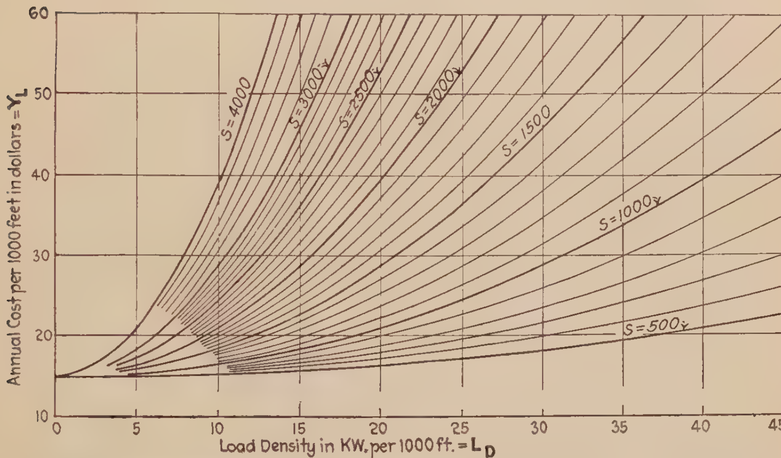

FIG. 45b.—Line cost curves. Annual cost per 1,000 ft. of 3 No. 4 secondaries including fixed charges and cost of lost energy.

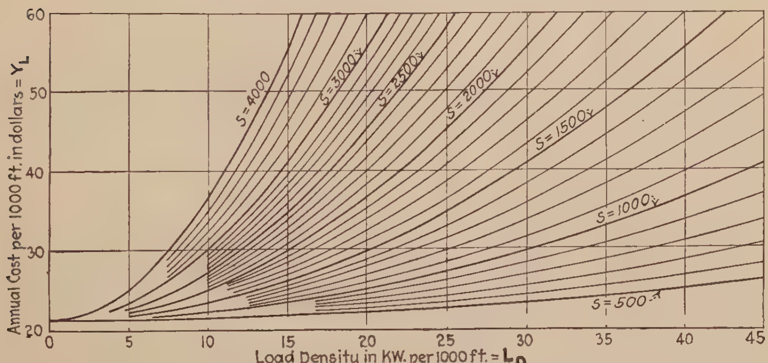

FIG. 45c.—Line cost curves. Annual cost per 1,000 ft. of 3 No. 2 secondaries including fixed charges and cost of lost energy.

All constants were evaluated and a curve plotted for each desired spacing—100-ft. intervals were used—showing the annual charges per 1,000 ft. in terms of the load density for each standard size of wire (see Fig. 45, *a*, *b*, *c*).

Transformer Cost Curves.—The third set of curves shows the annual cost on the transformer for any loading (see Fig. 46).

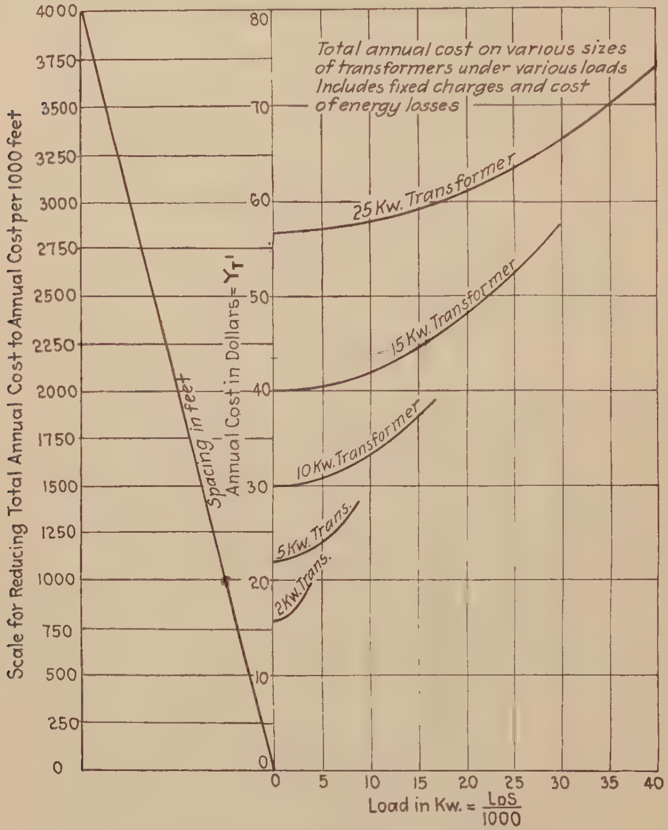

FIG. 46.—Transformer cost curves.

Annual cost of the transformer =

$$Y_{T'} = \frac{g_T}{100}\text{(cost of transformer + cost of lightning arrester + cost of installation) + inspection}$$

$$+ \text{cost of core loss}$$

$$+ \text{cost of copper loss}$$

$$= \frac{g_T}{100}\text{(cost of transformer + cost of lightning arresters.}$$

$$+ \text{cost of installation)}$$

$$+ \text{cost of inspection}$$

$$+ C_{e1} \times 365 \times 24 \times \frac{\text{core loss}}{1,000}$$

$$+ \frac{L_D{}^2 S^2}{E^2 \cos^2 \theta} R_T \times 365 \times t \times \frac{C_{e2}}{1,000} \qquad (71)$$

The equation for each of the standard sizes of transformers 2, 5, 10, 15 and 25 was developed and plotted. Since this curve shows total annual cost on a transformer and not cost per 1,000 ft. of installation, a scale was added on the diagonal at the left by use of which, with a pair of triangles, the cost per 1,000 ft. may be obtained by the principle of similar triangles.

$$\frac{S}{1,000} = \frac{Y_{T'}}{Y_T}$$

$$\therefore\ Y_T = Y_{T'} \times \frac{1,000}{S}$$

($Y_{T'}$ = total annual cost on a transformer)

(Y_T = annual cost of transformers per 1,000 ft. of installation) (see Fig. 47).

Hence by adding the diagonal scale at the left, Y_T may be obtained from $Y_{T'}$ as follows by the method of similar triangles. Draw a line from the value of $Y_{T'}$ obtained on the vertical scale to the value of S used, on the diagonal scale. Draw a parallel line through 1,000 ft. on the diagonal scale and where it intersects the vertical scale will be found the desired value of Y_T.

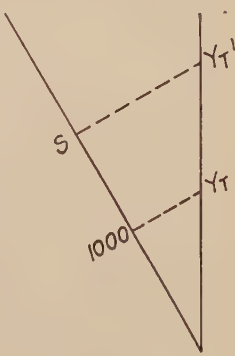

Fig. 47.

Cost of Replacing Transformers.—Two more items of cost are of interest to the designer and those are arbitrarily fixed by local conditions, the cost of changing the size of transformers in the same location and the cost of changing the location of a transformer. These will be practically constant for all sizes and may be determined in any case from local cost records.

Application of Practical Curves.—We are now ready to furnish the designer with the information necessary to test the relative economy of any two alternative designs. He first determines his wire size from a study of the theoretical and semi-practical curves. Then, going to the load curves he may determine his alternatives in transformer size and spacing. Assume that conditions point to the alternative of installing 10-kw. transformers at a long spacing, changing to 15-kw. at a shorter spacing after a certain period of years, or of installing the 10-kw. at the shorter spacing now and merely changing sizes at that time.

From our curves the exact cost per 1,000 ft. for each year under consideration may be obtained by using the correct loading and spacing and, at the proper time, adding the cost of either changing location or changing size. The total of the annual costs for each design gives the total cost over the period under consideration and a comparison of the totals shows exactly the relative economy of the designs over the whole period. These curves may be applied to any such problem since they are based not on the assumption of ideal conditions but cover any actual condition which might occur in practice. They can be used in cases where the transformer spacing cannot be uniform on account of local conditions of pole spacing, secondary length, and street and alley arrangement, a very usual case. When there is doubt about the wire size a study of the various possible combinations making use of these curves will soon determine the size for greatest economy. Similar curves can also be developed to suit other classes of problems such as concentrated loads, loads with characteristic variations different from those of the residence load used here, as in business districts, power loads, etc.

Example of Application of Practical Curves.—A concrete example of the use of the above curves may be helpful. Assume that tests on a district show a load density of 8 kw. per 1,000 ft., with No. 4 secondary wire already in place. Our load curves show for that loading and size of wire, 12.8-kw. load at 1,800 ft. spacing to keep within 3 per cent drop in voltage. We wish to provide for an increase in load which we will estimate may go to 15 kw. per 1,000 ft. in 6 years. For the present a 10-kw. transformer spaced at 1,400 ft. would care for the load while at 15 kw. per 1,000 ft. there would be required a 15-kw. transformer at 1,000 ft. spacing or a 25-kw. at 1,200 ft. In order to avoid too many changes we may space 10-kw. transformers at 1,000 ft., changing in 3 years to 15-kw. or we may put in 15-kw. transformers now at 1,500 ft., changing the location in 2 years to 1,000 ft. Other alternatives might be considered but these two will serve as an example.

For the first alternative, assuming uniform increase in load density of 1⅖ kw. per year.

First year.		
Line cost $- L_D = 8$ kw., $S = 1{,}000$........	\$16.00 per 1,000 ft. installation	
Transformer cost — 10 kw. at 8-kw. load....	32.00 per 1,000 ft. installation	
For year............................		\$ 48.00
Second year.		
Line cost $- L_D = 9\frac{3}{5}$ kw., $S = 1{,}000$......	16.30 per 1,000 ft.	
Transformer cost — 10 kw. at $9\frac{3}{5}$.........	32.70 per 1,000 ft	
For year..........................		\$49.00
Third year.		
Line cost $- L_D = 10\frac{4}{5}$ kw., $S = 1{,}000$.....	16.80 per 1,000 ft.	
Transformer cost — 10 kw. at $10\frac{4}{5}$........	33.70 per 1,000 ft.	
For year..........................		\$50.50
Fourth year.		
Line cost $- L_D = 12\frac{1}{5}$ kw., $S = 1{,}000$.....	17.30 per 1,000 ft.	
Transformer cost — 15 kw. at $12\frac{1}{5}$........	42.90 per 1,000 ft.	
Cost of changing size (10 kw. to 15 kw. on same pole).........................	7.00 per 1,000 ft.	
For year............................		\$67.20
Fifth year.		
Line cost $- L_D = 13\frac{3}{5}$ kw., $S = 1{,}000$.....	17.80 per 1,000 ft.	
Transformer cost 15 kw. at $13\frac{3}{5}$...........	43.60 per 1,000 ft.	
For year..........................		\$61.40
Sixth year.		
Line cost $- L_D = 15$ kw., $S = 1{,}000$.......	18.40 per 1,000 ft.	
Transformer cost — 15 kw. at 15..........	44.40 per 1,000 ft.	
For year..........................		\$ 62.80
Total for 6 years......................		\$338.90 per 1000 ft.
Second alternative		
First year.		
Line cost $- L_D = 8$ kw., $S = 1{,}500$	17.20 per 1,000 ft.	
Transformer cost — 15 kw. at 12-kw. load..	35.50 per 1,000 ft.	
For year..........................		\$ 52.70
Second year.		
Line cost $L_D = 9\frac{3}{5}$ kw., $S = 1{,}000$.........	\$18.00 per 1,000 ft.	
Transformer cost 15 kw. at $15\frac{4}{5}$...........	37.20 per 1,000 ft.	
For year..........................		\$ 55.20
Third year.		
Line cost $L_D = 10\frac{4}{5}$ kw., $S = 1{,}000$.........	16.80 per 1,000 ft.	
Transformer cost — 15 kw. at $10\frac{4}{5}$........	42.30 per 1,000 ft.	
Cost of changing location.................	20.50 per 1,000 ft.	
For year..........................		\$79.60
Fourth year—same as first alternative (less charge for changing size)		\$60.20 per 1.000 ft.
Fifth year—same as first alternative..............................		\$61.40 per 1,000 ft.
Sixth year—same as first alternative..............................		\$62.80 per 1,000 ft.
Total for 6 years....................................		\$371.90 per 1,000 ft.

A saving of \$33 per 1,000 ft. of installation, or approximately 10 per cent of the total cost over a period of 6 years by the first method thus demonstrating its economy. It is well to note that a large part of the difference in cost is due to the fact that in the first case the size of transformer is changed while in the other the location but not the size is changed. If a further refinement of the comparison is desired, interest may be considered on the yearly items up to the end of the period under consideration. Usually such refinement is not necessary however.

Conclusions.—A study of all these curves gives considerable aid in determining certain standards of design as well as the final particulars for any special problem. There also may be obtained a definite knowledge of the behavior of secondaries under various conditions of loading and operation. It is purposed here to take up each curve in detail, to bring out its characteristics and its possible use.

Most Economical Voltage Drop.—The curves on voltage drop show that the most economical condition varies inversely as the cube root of the wire size also inversely as the cube root of the load density.

For low load densities the economical drop is high but decreases rapidly, while at high loading the decrease is comparatively slow. It is clearly shown that the most economical voltage drop may be well under that allowable for good service for loads which may be often encountered in practice. Under the conditions and prices assumed in the present case the 3 per cent limit seems to have some justification by economy for ordinary loads.

Two conditions must be considered which might affect these curves, *i.e.*, the price of materials and cost of energy and the fact that here the transformer was considered just sufficient to carry the load while ordinarily, when designing for an increasing load, the transformers are underloaded. It is seen from the equation of the curves for economical voltage drop that the cost of copper does not affect this discussion. This is due to the fact that the annual cost is based on a unit of 1,000 ft. hence for any given price of copper the annual cost per 1,000 ft. of three-wire line is a constant no matter what the load. The cost of energy enters as an inverse factor to the $\frac{2}{3}$ power. Also it is a small element in the factor K_1, which is also to the $\frac{2}{3}$ power but in the direct ratio. Hence an increase in the cost of energy would increase both the numerator and the denominator but the latter slightly more than

the numerator, hence all the curves would be raised slightly. This effect would be small, however, for ordinary fluctuations. In the case of an increase in the transformer price there would be no change in the curves providing the increase were proportional to the size since the factor K_1 would not be affected by such an increase.

In ordinary design for an increasing load the transformer would be made larger than sufficient to carry the present load to allow for the anticipated increase. A study of the curves for the various components of the annual charge on a transformer and the equation resulting therefrom, $Y_T' = K_1 + K_2T$, will show that if they are developed with the transformer working below its rated loading, and if the percentage of underloading is kept the same for all sizes, the factor K_1 will be very little affected, the effect being similar to an increase in price proportional to size. Since this is the only part of this equation that enters into the equation for most economical voltage drop it follows that if a design could be limited to any given percentage of underloading throughout, the curves would still show the most economical condition of voltage drop.

Most Economical Transformer Spacing.—These curves for the most economical transformer spacing (Fig. 38) are derived from the same general expression for annual cost per 1,000 ft. as those for economical voltage drop. Hence, the same results might be expected from the use of either of these sets of curves with the exception that where the most economical spacing would give a maximum voltage drop of more than 3 per cent we have corrected it for that value making it such as to give 3 per cent.

These curves show for very low load densities, extremely high spacings which are probably much greater than it would be practicable to use since for such a distance and such light loads the effect of the non-uniform loading would be considerable. As is shown by the equation, the spacing for 3 per cent drop varies as the square root of the wire size while for greatest economy it varies as the cube root. It also varies inversely with the load density, to the square root in the first case, the $\frac{2}{3}$ power in the second. For ordinary loadings encountered in practice and the usual range of wire sizes it is seen that spacing of from 1,000 to 2,000 ft. is the most economical and practicable. For the higher loadings the most economical spacing decreases very slowly, remaining over 500 ft. up to high values of L_D.

Changed conditions would have a similar effect on these curves as on those for economical voltage drop, in the range of values for which the most economical voltage drop governs the spacing. That is, a rise in the price of energy would lower the curves slightly, the prices of wire and transformers would not have a noticeable effect. For the condition of underloaded transformers, if the proportion of underloading were fixed there would be slight change. In practical designing, however, when considering the amount of this margin in transformer capacity to be used, it might be found relatively more economical to use a transformer size somewhere near the theoretically most economical and obtain the margin in capacity by using a spacing less than the most economical spacing. This may have some advantage over using the most economical spacing, as shown by the curves, and a larger size of transformer than the most economical, when the design is to cover several years and the cost of changing sizes and locations is taken into account. Hence care must be used in placing too much dependence on the strictly theoretical values in practical design. The choice must be tested by the actual year to year costs as shown by the cost curves.

Most Economical Transformer Size.—The curves for the most economical transformer size simply show the size of transformer which will carry the load when the spacing is the most economical or just enough to give 3 per cent voltage drop. They have relatively less practical value excepting that it is from these and the spacing curves combined that the practical load curves are obtained.

Most Economical Wire Size.—The wire size is the first thing to determine in a design and must be chosen to cover long periods of increase in load as replacement of secondary wire is very costly. Hence for secondaries a standard must be chosen for installation in new work which will show good economy through the greatest range of conditions to be encountered. The curves seem to indicate clearly that under the conditions and prices assumed No. 6 wire should be used as a standard in all new work, in districts where ordinary residence lighting load is expected. The economy curve rises very rapidly at low densities up to about 20,000 cir. mil. or nearly No. 7 at about 7 kw. per 1,000 ft. From here the rise is less rapid but still considerable until it crosses the value of No. 6 wire at 15-kw. load density. The load density of 15 is a normal loading. It would not be advis-

able to use any size less than a No. 7 since the loadings at the smaller values are subject to such rapid increase. Even at No. 7 the economical load is fairly small (7 kw. per 1,000 ft.). On the other hand, the curve rises slowly after passing No. 6 and only reaches No. 5 at a loading of about 31 kw. and No. 4 at 40 kw. which are high densities and to be encountered only in special cases. It is interesting to note that for all values below a No. 6 wire the economical size is governed by 3 per cent voltage drop while above that the most economical drop governs, the curves crossing at 19-kw. load density.

Since the curves were figured at a low copper price, in case of an increase in price, the curves would be lowered, *i.e.*, a smaller wire size would be indicated for any particular loading. An increase in energy cost would slightly raise the curve, an increase in transformer price if proportional to size would not affect the discussion. Since the curves were figured on the assumption that the transformer spacing was the most economical and the size just equal to the load, a change in these conditions might affect the most economical wire size somewhat. A fixed proportion of underloading as above shown would have little effect but if different conditions of spacing were assumed, the design should be tested by use of the cost curves for various sizes of wire.

Semi-practical Curves.—The curves which we call semi-practical show a little more concretely the relative economy of installations with the various sizes of wire, in dollars per year per 1,000 ft. They show the actual annual cost for different types. The excessive cost of No. 2 wire for ordinary loads is clearly demonstrated being from $3.50 to $6.00 per year more than No. 4 for loadings up to 15 kw. per 1,000 ft.

When we go from the ideal size of transformer to practical stock sizes, still assuming the best spacings to be used, there are some conditions in which the relative wire economy is somewhat different. These curves also give an indication of transformer economy. It seems to be quite clearly shown that, under the assumed conditions, the use of large transformers such as 25 kw. is not justified except with very heavy loading, even considering the possible reduction in the number of transformers and hence in the core loss. The increase in the investment cost more than equalizes such saving.

Practical Curves.—The use of the cost curves in designing has already been explained. It may now be readily seen how a study

of the theoretical and semi-practical curves applied to any problem will give a basis upon which to formulate a design which can then be tested for actual economy by application of the exact costs to be expected. We can determine from this, in case of a new line, the size of wire and then the spacing and size of transformers which will care for several years of increase. The exact number of years will be determined by the rate of increase together with the economy of the design, including cost of changing sizes and locations. Or, in case of remodeling an old district, we start with a given size of wire which although perhaps not the most economical, will not justify the cost of change. We can then choose and space our transformers most economically with regard to that size of wire. In a special case where no increase in load is expected the theoretical curves will give exactly the design to use. In other cases where the loading, voltage, etc., are somewhat different, by proper substitution in the theoretical formulae, curves could be plotted which would apply to that particular condition.

General.—The curves given here should not be accepted for general application to design problems. The costs and conditions of loading used were of local derivation and apply only to the organization and the time for which they were obtained. Similar curves should be developed for the study of conditions in any other locality and they should be revised from time to time to meet changing conditions. These examples are given here merely to indicate the characteristics of such curves.

It is evident that no very simple means of correctly designing a distribution system in regard to transformers and secondary wire can be made available due to the many varying conditions encountered and the large number of factors to be taken into account.

The elements of good judgment and experience are as necessary in the solution of these problems as in any other problem of engineering. The object of this study has been to analyze and evaluate the factors of the design of a single-phase secondary system of the type considered, that lend themselves to such definite analysis and to present the results as aids in the application of good judgment and experience to the best possible solution of the problem. This problem has been dwelt on in some detail since it is thought that the methods used and the principles brought out are typical for a large number of such types of problems.

Secondaries for Scattered Load.—A very common question arising in rural districts where the load is scattered, is that of how far it is economical to extend secondary from a present transformer location to reach a new customer rather than to hang a new transformer. It is thought that this problem is of sufficient interest to warrant a brief mention here.

The comparison should be made, of course, on the basis of annual costs.

The annual cost on the installation in case the secondary is extended is

Y_s = annual charges on investment on line of length $D - D_s$
+ cost of I^2R loss on extended secondary
+ cost of increase in I^2R loss in transformer
+ cost of increased copper loss in transformer. (72)

Where D = distance from present transformer to new load,
D_s = distance from present transformer end of present secondary,

(It is assumed that the present transformer is large enough to carry the increased load.)

The annual cost in case the primary is extended and a new transformer used, is

Y_p = annual charge on investment on line of length $D - D_p$
+ I^2R loss in total length of primary D
+ total annual charges on new transformer including fixed charges and energy losses. (73)

Where D_p = distance from present transformer to end of primary.

In the problem considered, it is assumed that the cost of right-of-way, poles, crossarms, etc. would be the same in either case.

If the expressions for Y_s and Y_p are put in the form of equations in terms of the load, voltage, wire size, cost of energy, etc. and Y_s equated to Y_p, a solution may be obtained for D, the distance at which economy changes from a secondary to a primary extension.

The following expression was obtained using constants applying to a particular system.

$$D\left(.457\frac{W^2}{A}\right)\left(\frac{1}{E^2} - \frac{1}{E_1{}^2}\right) = .26(D_s - D_p)(.175w + .0125) - .0211\frac{W}{E^2}$$
$$\left(21.6\frac{L_D D_s{}^2}{A} + (W + 2W_1)R_{t1} - WR_{t2}\right) + K$$

Where W = new load in watts,
E = the secondary voltage,
E_1 = the primary voltage,
w = weight per foot of conductor (primary and secondary assumed same size for small load),
W_1 = present load on present transformer,
R_{t1} = equivalent resistance of present transformer,
R_{t2} = equivalent resistance of new transformer,
K = fixed charges and annual cost loss cost on transformer,

If both primary and secondary were two No. 6 wire and E_1 = 4,600, E = 112 — the equation becomes

$$1.382D\left(\frac{W}{1{,}000}\right)^2 = 8.35(D_s - D_p) - 1{,}680\left(\frac{W}{1{,}000}\right)\left(\frac{0.823}{10^6}L_DD_s^2 + \left(\frac{W}{1{,}000} + \frac{2W_1}{1{,}000}\right)R_{t1} - \frac{W}{1{,}000}R_{t2}\right) + 1{,}000\,K$$

In the limiting case, where $D_p = D$ and $D_s = 0$ the cost of primary extension would be a minimum and of secondary extension a maximum. If $T_1 - T_2 = 2$ kva. and T_1 is fully loaded

$$D = \frac{18{,}550 - 4{,}180\left(\frac{W}{1{,}000}\right)}{1.382\left(\frac{W}{1{,}000}\right)^2 + 8.35} \tag{74}$$

This indicates the distance to which it is economical to carry any load, W, on secondary.

Since, for 3 per cent voltage drop with the above conditions

$$D = \frac{417}{W/1{,}000}$$

Solving simultaneously

$$\frac{W}{1{,}000} = .2\,\text{kw.}$$

This indicates that, for any load over 200 watts, economy need not be considered if the maximum allowable voltage drop is fixed at 3 per cent, since it is economical to carry such a load on secondary to any distance at which the voltage drop is 3 per cent or less.

For loads less than 200 watts, Eq. 74 may be plotted in a curve if desired.

The above solution has been given very briefly and all but the chief steps omitted. It indicates the method which can be

followed in studying a number of similar problems. The rule established in the particular case shown has been found very useful in laying out rural extensions.

Space will not permit the elaboration of more problems of single phase secondaries. Among the others often encountered are the following: economy of replacing a small secondary with a larger size instead of hanging an additional transformer (see Chap. X on "Reconstruction Problems"); replacing wire larger than the economical size with a smaller size; the economical size of secondaries and arrangement of transformers for electric range loads; economy of leaving dead wire in place if it is to be utilized later; and many others.

CHAPTER XIV

POWER SECONDARIES

Power Secondary vs. Separate Transformers—Economical Size of 3ϕ Secondaries

The power secondaries on any system are, as a rule, by no means as extensive as the lighting secondaries. In districts where the power load is very heavy, it is very often advisable to supply each customer from separate transformers. In districts where the power load is scattered, the distances are usually too great to carry more than a very few customers on one secondary. There are, of course, many cases where several small or medium sized loads are grouped in a relatively small area, such as a number of small factories or a block of stores. For such conditions, it is usually practicable to use a few large transformer installations with power secondaries. While the proportion of the total system investment represented in power secondaries in relatively small, nevertheless the amount of load handled on any one installation is usually large compared with that on a lighting secondary. Hence, although a consideration of their economy, as a general proposition may not seem so important, in individual cases it may be quite profitable.

It very rarely happens that the load on a power secondary is so arranged that it may be considered as a distributed load, either uniformly or in accordance with any other definite law. Neither does it often occur that power secondaries can be made continuous and the transformers spaced as desired. Hence, the problem is usually one of concentrated loads of a given size to be transmitted a definite distance. There must be a study of each particular case rather than of the type of installation in general.

Two kinds of problems are quite commonly met with in connection with power secondaries. It must be decided, for any case of a small or medium sized load, whether it is preferable to carry it on a separate transformer or tap it to a power secondary, providing one is available or can be installed. If it is to be thus handled, the most economical size of conductor should then be determined.

Regulation and Continuity of Service Important.—The deciding factor is quite likely to be some other consideration than that of economy only. It must first be determined whether the load can

be carried from the present installation, with a reasonable sized secondary, without exceeding the allowable voltage drop. Oftentimes, a present transformer installation can be moved to a new location and the secondaries rearranged to accommodate such additional loads. Even though it is found easily possible to carry the load in this way there are some cases where the importance of continuity of service may indicate that a separate installation is advisable. Any trouble on one customer's service or in the transformer would disable the services of all other customers. Hence, any customer whose service is especially subject to interruptions, or any one whom an interruption would seriously discommode should be given separate transformer installations. For the same reason, it is well not to concentrate too many power services on one secondary, especially services to manufacturing plants.

Economy of Secondary Instead of Separate Transformers.—If it has been decided that the load in question can and should be carried on secondary, it still remains to determine whether such an installation is economical. If the annual charges including cost of energy loss on the secondary, necessary to reach the customer, is greater than the annual charges on a separate transformer installation, the latter should be used. If the whole installation is to be rearranged, the annual charges on the whole cost of making the change must be included. For small loads or loads near a present transformer, a secondary installation is, in most cases, more economical. The cases to be questioned are those where heavy secondaries of considerable length are necessary.

For example, suppose a new load of 25 hp. is to be served, which is 800 ft. from a present installation. In order to get proper regulation, it will be necessary to string No. 0 secondaries the whole distance. The transformer which at present is a 75 kva., well loaded, must be changed to a 100 kva. It will be assumed that the necessary poles will be the same for either a secondary or a primary extension to the customer. To determine the most economical installation, we must compare the annual costs of the two alternatives. These are made up as follows:

Secondary Installation.—

1. The annual charges on 800 ft. of three No. 0 secondary, including cost of conductor, crossarms, pins, insulators, and labor cost for installing.

2. Annual cost of energy losses over the new secondary.

3. Annual charges on cost to change transformers.

4. Annual charges on increase in transformer investment from a 75 to a 100 kva.

5. Annual cost of increase in transformer energy losses.

Primary Installation.—

1. Annual cost on 800 ft. of primary, unless the present primary extends past the new load.

2. Energy losses on the primary.

3. Annual cost on a new transformer installation of 25 kva., including losses.

Some such figures as the following may be obtained:

Secondary installation		Primary installation	
No. 1	$ 39.00	No. 1	$ 15.00
No. 2	32.00	No. 2	2.50
No. 3	2.00	No. 3	118.00
No. 4	35.00		
No. 5	14.00		$135.50
Total	$122.00		

The secondary installation will have advantage of $13.50 per year which at 15 per cent represents a capitalization of $90.00 If the primary is already in place, the advantage would be slightly with the separate installation, although the difference in cost is small.

Economical Size of Secondaries.—When it has been decided upon that a load is to be carried on secondary, either after some such consideration as the above, or when, with a separate installation, in order to locate the transformer conveniently, it is necessary to string a few spans of secondary from the transformer to the service, there still remains the question of the most economical size of conductor to be used. Let us assume for this example small 3 ϕ power installations for which the load is considered continuous during the time of operation.

Equation of Annual Cost—

$$\begin{array}{r}\text{Annual cost on } 3\phi \text{ secondary} \\ \text{per 100 ft.}\end{array} = \left[\begin{array}{l} g(\text{cost of wire} + \text{cost of stringing}) \\ +g(\text{cost of poles, fixtures and guys}) \\ + \text{cost of energy loss.}\end{array}\right] \quad (75)$$

Where g = per cent interest, taxes and depreciation
= 13 per cent for wire.

Assume poles, fixtures and guys the same for all cases and neglect cost in making comparison. No doubt with the heavier

sizes of wire, additional cost will be found necessary for heavier cross arms, insulators, etc. and additional guying. The study could be made so as to include these, as was done in the case of primary lines in Chap. XI, but in this case this factor will be left out of the computations. In using the resulting curves it can be kept in mind and will have the effect of increasing slightly the economical load on those sizes.

Annual charge for energy loss (3ϕ) =

$$I^2R \times 365 \times t \times \frac{C_e}{1{,}000} = .365 I^2RtC_e$$

$$I^2R = \frac{kw^2 \times 10^6}{3E^2 \cos^2\theta} \times 3Dr = 10^6\, Dr \frac{kw^2}{E^2 \cos^2\theta}$$

$$\text{Annual charge (energy)} = 365{,}000\, Dr \frac{kw^2}{E^2\cos^2\theta} tC_e$$

In the above t = equivalent hours per day,

If t_w = equivalent hours per week which is assumed in this case to be hours per week which motor runs and power is given in horsepower.

$$E = 220 \text{ volts; } \cos\theta = .80$$

$$\text{Annual charge} = 52{,}000\, Dr \frac{\overline{HP}^2 \times (.746)^2}{E^2\cos^2\theta} t_wC_e$$
$$= .933 Drt_wC_e\overline{HP}^2$$

D = distance one way in hundreds of feet.

r = resistance per wire per 100 ft.

Annual charge per 100 ft. (energy) = $.933 rt_wC_e\overline{HP}^2$

The cost of the conductor per 100 ft. was determined from the number of pounds per 100 ft. for different sizes, the cost of ties, and the proportional charge for freight and injury to returned reels. This was put in the form $K_mC_{cu} + K_n$, so that the effect of a change in price of copper could be studied. The labor cost was determined for average conditions and reduced to a cost per 100 ft. for each size. The proper overhead loading percentage was added to both material and labor. Combining these two charges, the expressions for annual charges on the conductor in place for each size were determined as shown in Table 17, p. 179, column 2.

Column 3 of the same table gives the resistance per 100 ft. of the various sizes of conductors and column 4 the corresponding expression for annual charge for energy loss, found by inserting the proper value of r in equation above. The sum of columns 2 and 4 would give the total annual charges per 100 ft. for each size of conductor.

Equations of Equal Cost.—If the expression for the total charges on one size of wire is equated to that for any other size, the resulting equation would represent the conditions for which there is no economical advantage of one size over the other. This is done for each adjacent size in the table and the resulting expressions are given in column 5.

If we consider two prices of copper, the results for any intermediate price can be interpolated. In the table below the expressions for equal annual cost are given with the copper price substituted, using prices of 20 cts. and 30 cts. per pound.

TABLE 18

Size of Wire		Size of Wire	If C_{cu} = .30	If C_{cu} = .20
6	to	4	$t_wC_eHP^2 =$ 55.99	$t_wC_eHP^2 =$ 39.04
4	to	2	$t_wC_eHP^2 =$ 165.77	$t_wC_eHP^2 =$ 113.47
2	to	0	$t_wC_eHP^2 =$ 465.00	$t_wC_eHP^2 =$ 317.20
0	to	00	$t_wC_eHP^2 =$ 788.50	$t_wC_eHP^2 =$ 547.50
00	to	000	$t_wC_eHP^2 =$ 1276.80	$t_wC_eHP^2 =$ 889.80
000	to	0000	$t_wC_eHP^2 =$ 1784.40	$t_wC_eHP^2 =$ 1214.40

Value of t_wC_e.—In previous chapters the variation of the cost of energy with the load factor and hence with the equivalent hours has been explained. In "Appendix *A*" the method of approximating the value of t_wC_e is carried out. In this case, the loads are considered small, consisting of not more than two or three motors, which would give a flat load curve. The value of equivalent hours per week, t_w, would then be practically equal to the average number of hours per week which the motors are run. In case of loads of a different character, a more accurate determination of t_w would be necessary. The values of t_w for any load factor, and the corresponding values of t_wC_e used are given below.

TABLE 19

t_w	LOAD FACTOR	t_wCe
.0	.0	.0
16.8	.10	.467
33.6	.20	.619
50.4	.30	.756
67.2	.40	.894
84.0	.50	1.015
100.8	.60	1.139
117.6	.70	1.258
134.4	.80	1.358
151.2	.90	1.450
168.0	1.00	1.543

TABLE 17

Size	Annual charge on construction	Resistance 100 ft. $= r$	Annual charge on energy loss	Equation of equal cost
3 No. 6	$3 \times .13(14.4C_{cu} + 1.33) = 5.619C_{cu} + .519$	.0403	$.0376t_wC_e\overline{HP}^2$	$.0140\ t_wC_e\overline{HP}^2 = 2.376C_{cu} + .072$ $t_wC_e\overline{HP}^2 = 169.5C_{cu} + 5.14$
3 No. 4	$3 \times .13(20.5C_{cu} + 1.512) = 7.995C_{cu} + .591$	.0253	$.0236t_wC_eHP^2$	$.0088\ t_wC_e\overline{HP}^2 = 4.605C_{cu} + .078$ $t_w\overline{HP}^2 = 523C_{cu} + 8.87$
3 No. 2	$3 \times .13(32.3C_{cu} + 1.717) = 12.60C_{cu} + .669$	.0159	$.0148t_wC_e\overline{HP}^2$	$.00528t_wC_e\overline{HP}^2 = 7.80C_{cu} + .114$ $t_wC_e\overline{HP}^2 = 1478C_{cu} + 21.6$
3 No. 0	$3 \times .13(52.3C_{cu} + 2.01) = 20.40\ C_{cu} + .783$	.0102	$.00952t_wC_e\overline{HP}^2$	$.00197t_wC_e\overline{HP}^2 = 4.74C_{cu} + .129$ $t_wC_eHP^2 = 2410C_{cu} + 65.5$
3 No. 00	$3 \times .13(64.4C_{cu} + 2.34) = 25.14C_{cu} + .912$	.0081	$.00755t_wC_e\overline{HP}^2$	$.00158t_wC_e\overline{HP}^2 = 6.12C_{cu} + .183$ $t_wC_e\overline{HP}^2 = 3870C_{cu} + 115.8$
3 No. 000	$3 \times .13(80.2C_{cu} + 2.804) = 31.26\ C_{cu} + 1.095$	.0064	$.00597t_wC_e\overline{HP}^2$	$.00121t_wC_e\overline{HP}^2 = 6.90C_{cu} + .090$ $t_wC_e\overline{HP}^2 = 5700C_{cu} + 74.4$
3 No. 0000	$3 \times .13(97.9C_{cu} + 3.034) = 38.16C_{cu} + 1.185$	.0051	$.00476t_wC_e\overline{HP}^2$	

The curve, Fig. 48 is plotted from the above figures, giving the value of $t_w C_e$ for any value of t_w.

Curves for Economical Conductor Size.—The two sets of curves given in Fig. 49 and Fig. 50, can now be plotted from the equations in Table 18, using as coordinates the average number of hours of operation per week and the load in horsepower (the load is given in horsepower for convenience in use with loads consisting of one or two motors which are so rated). The curves are similar to those given for "Power Lines" in Chap. XI. The conductor sizes given pertain to the areas between the curves,

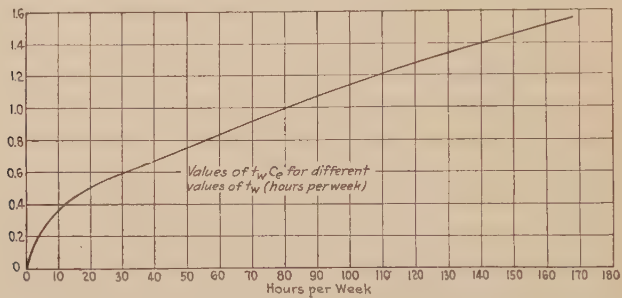

FIG. 48.—Values of $t_w C_e$ for different values of t_w (hours per week).

since the curves themselves are the loci of points of equal cost. For example (using 20-ct. copper) three No. 2 secondary, is more economical than three No. 4 for a 20-hp. load, if it operates more than 6½ hr. per week. It is less economical than three No. 0 if the 20-hp. motor operates more than 54 hr. per week. Similarly for a load operating 4 hr. per week, three No. 0 is the most economical secondary for loads between 21½ hp. and 28½ hp. If higher-priced copper is being used, the corresponding loads and hours of operation will be larger, as shown by Fig. 50. The distance which the point representing a given load lies from either boundary curve is an indication of the amount of economical advantage of the size of wire shown, over the next adjacent size.

Naturally, these curves apply only to concentrated loads of the character assumed. Similar curves may be developed for any type of load desired by the inclusion of the proper factors in making up the equation. For loads which are not concentrated, the secondary may be studied by dividing into sections. Curves for large power loads can be similarly developed. In that case, however, it is probably better to designate the load in kilowatts

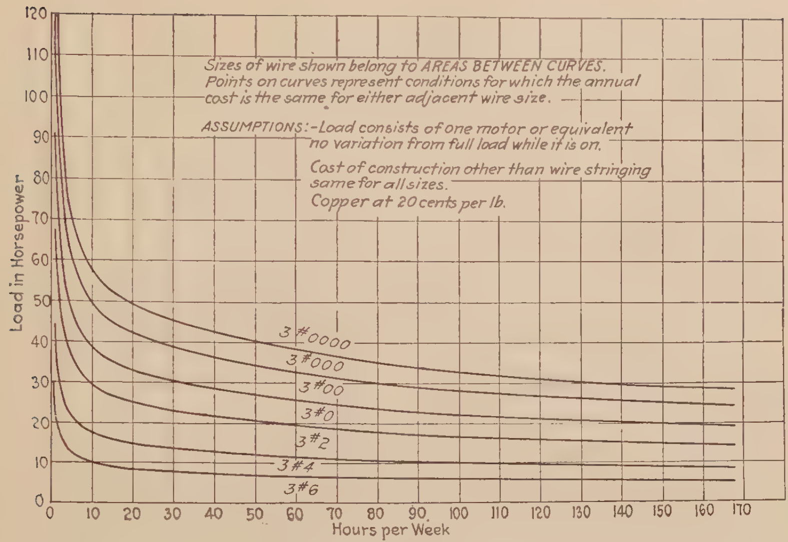

FIG. 49.—Curves showing most economical size of wire for three-phase secondaries.

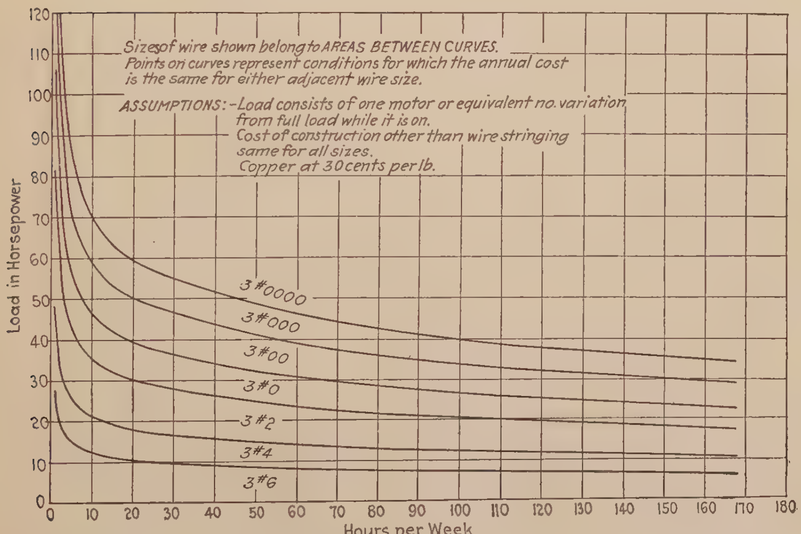

FIG. 50.—Curves showing most economical size of wire for three-phase secondaries.

and use values of t = equivalent hours per day, as was done in the case of power primaries, Chap. XI.

Problems in Power Secondaries Comparatively Simple.—The problems relating to power secondaries are comparatively simple. With good cost data on the installation of such secondary and transformers, load curves for power secondaries as shown in Chap. VII, and curves for economical conductor size, nearly all such problems as are discussed in this chapter may be readily solved. Power secondaries may be, of course, a relatively less important part of the system, than transmission lines, primaries or lighting secondaries. Notwithstanding this fact, however, the study of their economy should not be overlooked.

CHAPTER XV

UNDERGROUND LINES

Voltage—Cable Size—Route—Number of Ducts in a Duct Line—Arrangement of Ducts and Cables

The major part of this book has been devoted to the problems connected with overhead lines. This was done not because the economic study of underground lines is any less important than that of overhead. The opportunities for effecting economies are just as great, or perhaps greater, on underground lines on account of the greater cost of construction. It is a fact, however, that on most of the central-station systems in this country, the underground lines are of small extent as compared with the overhead. Overhead work is preferred where possible, underground being used chiefly in congested down-town areas and for transmission lines where high voltage is not permitted by the municipality, or is considered unsafe. In some other countries, the case is quite different, underground work predominating. In any case, the principles employed in solving the problems of underground lines are the same as those used for overhead which have been explained and exemplified. Only the conditions of the individual problems are different. Some of the affecting factors and special problems relating to underground lines will be taken up in this chapter.

The necessity rarely arises to make a choice, on the basis of economy, between an underground installation and an overhead. Overhead, as a rule, costs only a fraction of the amount necessary to install underground for the same load. An overhead transmission line, for example, can be built for between $3,000 and $4,000 per mile to carry the load which would require an underground installation costing $15,000 to $18,000 per mile. Overhead also has the advantage of greater flexibility when changes are necessary to accommodate increases in load. The choice of underground is usually based on considerations of necessity (in congested areas), safety, sightliness, etc.

The nature of the construction on underground lines makes the necessity for standardization and for the best possible workmanship very evident. The comparative inaccessibility and

the high cost of installation make repair work very expensive. True economy lies in using all means possible to reduce such repair work to a minimum.

Problems Similar to those of Overhead Lines.—A great many of the problems of underground lines are very similar to those already discussed for overhead. There is, for example, the same general classification into transmission lines, primaries and secondaries, with the special questions arising with each. The economy of any installation should be similarly studied with respect to voltage, voltage drop, conductor size, transformer size and location, most economical route, etc. The solution of these problems with respect to underground lines is limited by their character. Inflexibility and high cost of construction make it necessary that original installations be designed with sufficient thought toward probable future conditions, even at the expense of apparent present economy in some cases. There are some other problems concerning the construction itself, which apply only to underground lines, such as the number and arrangement of ducts in a run, and of cable in the ducts, etc. These will be taken up after some of the questions of voltage, conductor size, route, etc. have been briefly considered.

Voltage.—The determination of the most economical voltage is rarely dependent on the economics of the underground system alone. In transmission, the voltage is limited by the type of cables available. At present, about 33,000 volts is the practicable limit, although higher voltages are being considered and will probably be used in the near future. For power lines, one or two standard voltages are usually chosen for the whole system. While the economics of the underground lines should be considered in this choice, it is by no means the only factor. A study of the system as a whole is necessary.

Conductor Size.—A new element is introduced in the study of the most economical cable to carry any load or the most economical load for any cable. The limiting factor in this case is usually the current carrying capacity of the cable. This is governed by the heating of the cable. It depends not only on the size of conductor and on the insulation but on the location of the cable in the duct run, the number of ducts, the condition of the surrounding soil and the shape of the curve of the load on the cable itself and on the other cables in adjacent ducts.

The question of the heating of cables has been attracting a

great deal of attention recently and a number of articles on this subject have appeared in the *Journal* of the A. I. E. E. and other publications. The study is still in the making and, while much valuable data has been collected, there is still much to be done. At present there is no generally accepted standard on such matters. It is safe to say that in practically all cases of ordinary loads, the apparent economical load for a cable, basing the figures on the normal life of that cable, will be considerably more than the safe current carrying capacity, if the cable is to fulfill its

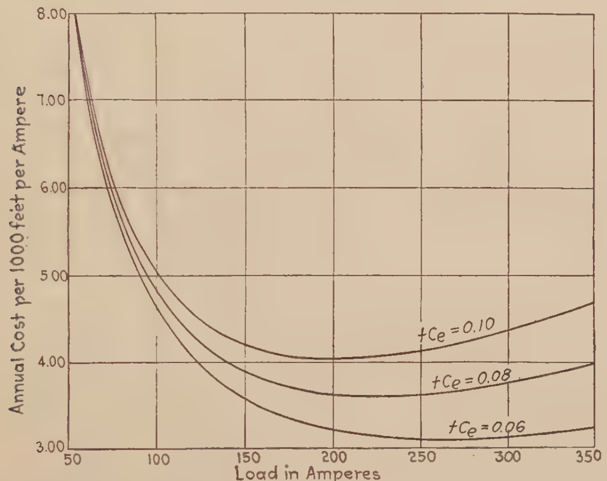

Fig. 51.—Annual cost per 1,000 feet per ampere transmitted No. 00 underground cable 23,000 volts.

normal life. For example, Fig. 51 shows the annual cost of transmitting loads over No. 00 underground cable at 23,000 volts for various values of equivalent hours. In making the computations, depreciation was figured on the basis of a 20-year life for the cable. It is seen that the most economical loadings on that basis would be between 200 and 300 amp., depending on the load factor (or the equivalent hours). Figure 52 gives the average allowable load for such a cable in various combinations with similar cables, based on the heating of the cable. It shows that the safe loading in duct runs of the usual size is in the neighborhood of only 100 amp., or about 50 per cent of the most economical load, as shown by Fig. 51. It might be thought

possible to run the cable at a higher average load than that given as the safe load, letting the increased depreciation due to the shortening of the life of the cable be balanced against the increased economy of operation. Not enough data is available on the extent to which a cable is damaged by an overload to warrant any figures on this subject. It is quite probable that the damage done would be greater than the economy effected in most cases. In the example represented by Fig. 51, if the load were such as to reduce the life of the cable to 10 years, the apparent most economical load at tC_e = .10 would be about 225 amp. However, if a load of 140 amp. or greater could be carried without reducing the life of the cable to less than 10 years, the annual

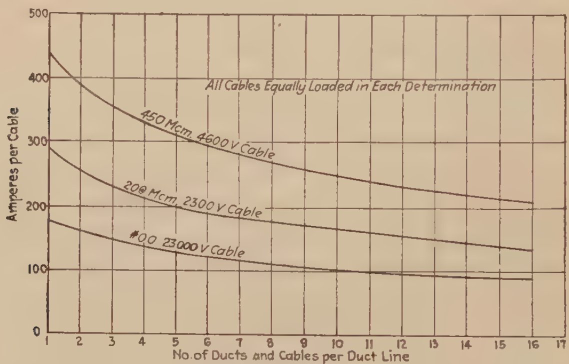

FIG. 52.—Curves showing current carrying capacity per cable vs. number of cables per duct line for three conductor paper and lead cables.

cost per ampere would be less than that at 100 amp. with a 20-year life for the cable. If a cable is to be run at a load greater than that considered as perfectly safe from a standpoint of heating, there is the further consideration of increased dielectric losses and increased resistance of conductor. The problem has great possibilities when more data becomes available.

Figure 51 also shows how the most economical load increases as the load factor, and hence the value of equivalent hours, decreases. Since cable heating has somewhat of a cumulative effect, it is generally possible to carry higher loads at low than at high load factors. The safe load will, therefore, bear somewhat the same relation to the most economical load, in any case. Hence it appears that, in most conditions met with in practice,

the load carried will be governed by the capacity of the cable rather than by economy as usually considered, although, of course, economy is realized by not overloading a cable to the point of injury.

Economical Route.—The most economical route for an underground line may be an important consideration in its layout. On account of the high construction cost, the use of the shortest possible route is even more important in this case than with overhead lines. Other things being equal, this might often point to the use of private right-of-way. The relative inaccessibility of such lines on private property, the possibility of interference by future buildings, etc., unless the property is bought outright, and the difficulty of draining manholes in many cases, generally makes it preferable to keep such lines on the public highway. The number and location of manholes necessary may have considerable bearing on the choice of a route. In runs of only a few ducts, the cost of manholes is a large proportion of the total and hence is relatively very important. In runs with a large number of ducts the manhole cost is less important compared with the total cost of the line, but in any case, the route requiring the fewest manholes, other things being equal, has considerable advantage.

Secondary Distribution.—The problem of underground secondary distribution is quite different from that on overhead lines. Transformer locations are limited to the manholes which in turn must be spaced for convenience in installing and maintaining the cable. The secondaries and services must be designed to transmit the required load with ample provision for future contingencies. The changing of secondaries of services or of transformer locations is a considerably more serious matter than on overhead lines. In general the design of an underground secondary system is a problem requiring a great deal of care and good judgment in predicting future loads and providing for them in an economical manner.

Number of Ducts in a Duct Line.—There are a number of problems relating to the construction and arrangement of ducts and cables which are worthy of attention. An example will be given here of a method of determining the most economical number of ducts which should be placed together in one duct run. While it would not always be practicable to limit the number of ducts to the figures which might result from such a

study, the knowledge thus gained would at least be an aid in making the choice. For simplicity, in this example it will be assumed that the ducts are all filled, the cables are all of the same size, and the loads carried on all the cables are of equal amounts and identical characteristics. Naturally such a condition would rarely be found in practice. However, if the method of attacking the problem is once established, it can be extended to cover other cases of dissimilar cables and loads.

The basis for determining economical conditions is the annual cost per ampere transmitted over the line as a whole. The equation for annual cost of the line is made up as follows:

Annual cost per 1,000 ft. = g (cost of 1,000 ft. of duct line, of n conduits, in place)
$+ g$ (average cost per 1,000 ft. for manholes complete with sewer connections)
$+ g$ (cost per 1,000 ft. of n cables installed)
$+$ cost of energy losses per year over 1,000 ft. of n cables. (76)

Cost of Duct Line.—The cost of installing a duct line is made up of:

1. Cost of excavating, backfilling, paving, etc. which is nearly proportional to the width of the duct line. If the cross-section of the duct is square, or nearly so, the width is practically proportional to the square root of the number of conduits, $\sqrt{n}$.

2. Cost of materials used and labor of installing which is practically proportional to the number of conduits, n.

3. Cost of transportation, tools, water connections, etc. which is practically the same for all sizes.

The total cost of the duct in place is then represented approximately by the expression $K_1 + K_2\sqrt{n} + K_3 n$ where K_1, K_2 and K_3 are constants to be determined from actual field costs on several jobs.

Cost of Manholes.—The cost of building a standard manhole is usually easy to determine. The cost of drainage and sewer connections will vary, of course, with different conditions encountered. An average figure must be assumed. By determining the average number of manholes per 1,000 ft. of duct, the

annual charges per 1,000 ft. for manholes complete may be easily computed.

Cost of Cables.—The cost per foot of cable is readily determined from current prices and average labor costs of the system under consideration.

Energy Losses.—The energy loss depends on the current

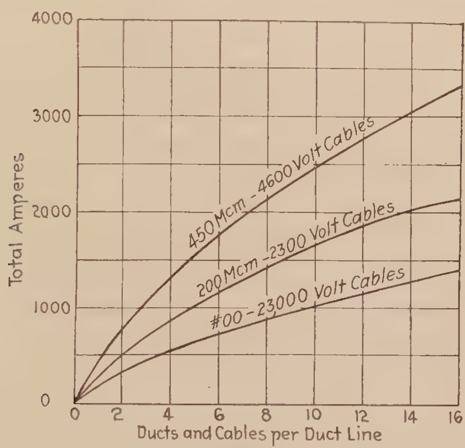

FIG. 53.—Curves of total current carrying capacity of duct lines vs. number of ducts for various 3 conductor paper and lead cables.

carried. If the number of conduits in the duct is large, the allowable current per cable will be smaller than in a duct run of few conduits on account of the increased heating effect. A study of the current carrying capacities of lead covered cables under various conditions is given by Ralph W. Atkinson, in the *Journal* of the A. I. E. E. for September, 1920. By use of the figures and charts given in that paper, the variation of the allowable current on a cable with the number of conduits in the duct line was derived, for three types of cables, and plotted in Fig. 52. Figure 53 is derived from Fig. 52 and gives the total allowable number of amperes carried on the duct line as a whole, for any number of conduits.

If I_c = the allowable current per cable,

and r = the resistance per 1,000 ft. of one conductor of one cable. The annual cost of energy losses per 1,000 ft.

$$= n\left(3I_c^2r \times t \times 365 \times \frac{C_e}{1{,}000}\right)$$

g.—The value of *g*, *i.e.*, the per cent of interest, taxes, depreciation, etc. will vary for the different classes of material. Such figures as the following can be assumed for any problem.

Interest		7 per cent
Taxes		2 per cent
Depreciation, etc.,	on ducts	2 per cent
	on manholes	5 per cent
	on cables	5 per cent

Total Annual Cost.—After the various individual costs, as indicated above, have been evaluated, they may be combined to give the total annual cost. Figures 54, 55, and 56 show, for

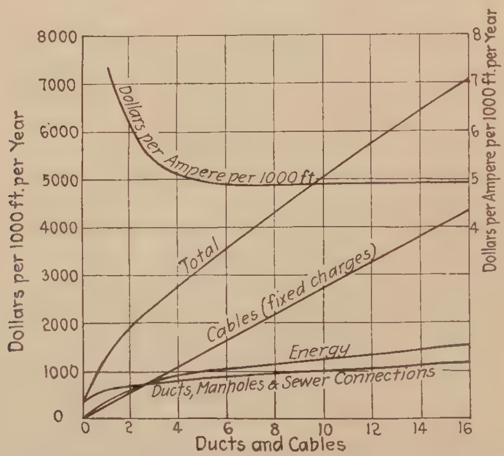

FIG. 54.—Curves showing annual costs per 1,000 feet vs. number of ducts and cables for No. 00 3-conductor cables (23,000 volts).

the three different types of cable, the variation of the cost of the different items given above as a function of the number of ducts in the line. The numerical values, of course, apply only to the local conditions for which they were derived. The curve for total annual cost is a summation of the separate charges. If the total annual cost for any given number of ducts and cables is divided by the total allowable current carried, as given by Fig. 53, the cost per ampere is obtained. This is given in the upper curve on Figs. 54, 55 and 56.

Results Shown.—The curves indicate clearly the high cost of less than four ducts in a run. For larger lines, the point of econ-

omy is not so plainly shown. There appears to be comparatively little difference in economy between duct runs of from 4 to 16

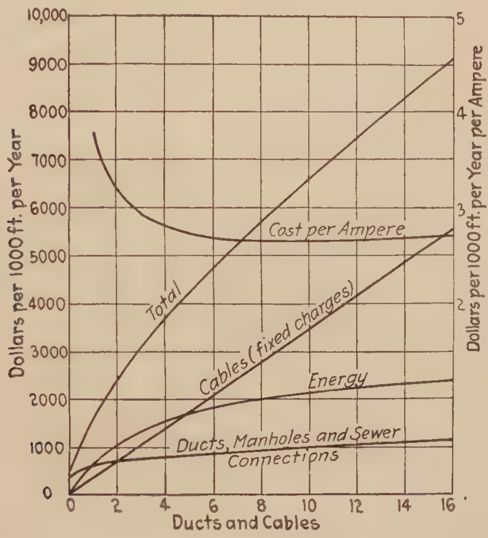

FIG. 55.—Curves showing annual costs per 1,000 feet vs. number of ducts and cables for 450 cm. 3-conductor cables (4,600 volts).

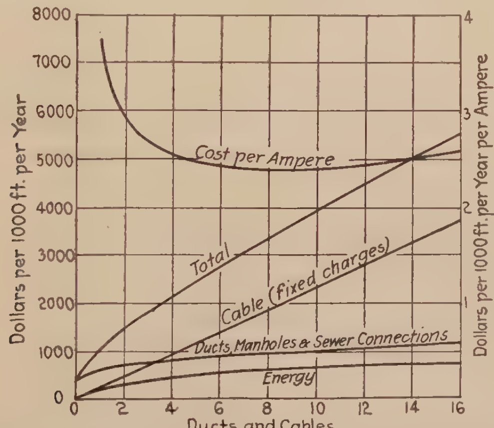

FIG. 56.—Curves showing annual costs per 1,000 feet vs. number of ducts and cables for 200 M cm. three conductor cables (2,300 volts).

ducts. The curves for 2,300-volt cable, Fig. 56, indicates a minimum point at about eight ducts, but the curve is fairly flat. The reason for the small difference in cost shown is due to the fact

that as the number of ducts increases the allowable current per cable decreases. It would seem to be indicated, in this case, that there would be considerable advantage in not building duct runs to provide too far into the future. If the total cost per ampere of a six- or eight-duct run is no more than that of a 16, it would be better to build the smaller size and when that is filled, build another of the same size, thus saving the investment on empty ducts for a considerable period. Of course, if different cost figures were used or different assumptions as to cable sizes and loadings were made, the points of greatest economy might be more pronounced. Hence these curves must be considered as examples of the method only and not for general application.

Arrangement of Ducts.—Another detail of construction which might offer a profitable field for investigation is the arrangement of ducts in a duct run. It is realized that the practical difficulties in construction may be the deciding factor in this matter. However, it must be kept in mind that the carrying capacity of a cable depends a great deal upon its location with respect to other cables. The center cable in a nine duct run arranged in a square will have considerably less capacity than the outside cables, while the corner cables will have more capacity than those between (Fig. 57). This is due to the relative capability of heat dispersion of the various positions. It would be an interesting problem to determine how much extra expense would be justifiable in order to use some arrangement which would accomplish better heat dispersion. The problems of increasing the current capacity of a duct line by using the center ducts for some form of a cooling system or of flooding the runs are similar. No definite data is at present available on these questions.

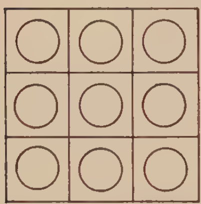

Fig. 57.—Arrangement of ducts in duct run.

Arrangement of Cables in a Duct Line.—The question of the relative heating of cables leads to the problem of the proper arrangement of cables in a given duct run. It rarely happens that the cables are all of the same size or kind or carry similar loads. The diversity between loads may have considerable bearing on the most economical arrangements of the cables. Obviously a cable carrying a heavy load with a high-load factor can operate more efficiently if placed in an outside duct where the heat can be more rapidly dissipated. Similarly, if a cable carry-

ing night lighting load only is placed in an interior duct, surrounded by ducts containing cables with day power loads only, and the peak loads do not overlap, the lighting cable can be operated at a higher load than would be considered safe with all cables similarly loaded.

Possible Savings Large.—The possibilities for economic investigations on underground lines are large. The loads carried are fairly heavy as a rule, since underground work is usually done in congested districts. The construction cost is high. Savings of a few per cent mean a relatively large amount of money in the long run. The examples presented above and the other problems mentioned are intended merely as suggestions for study along this line. The available data on heating of cables, allowable current carrying capacity, ageing, etc. is as yet so unreliable that conclusive solutions of any problem are difficult.

CHAPTER XVI

THE SYSTEM AS A WHOLE

There still remain a great many problems, both large and small, which are continually confronting engineers in charge of distribution systems. Some of these are special cases but can be solved by an adaptation of the principles indicated herein. Others apply to other parts of the system than the distribution lines, such as to details in the design of the generating station or the substations. No attempt has been made to cover such questions specifically, although the general methods could be applied. There are still other problems which deal with the system as a whole. While it is not within the province of this work to discuss such questions at length, a few of the most important will be mentioned here.

A considerable amount has been published at different times and at various places about the proper location of a generating station. Theoretically, it should be located in such a way, with respect to the loads to be carried, that the total annual cost on the completed system will be a minimum. This does not necessarily mean that the best location is at the center of gravity of the loads as is sometimes stated. If we consider two equal loads, one operating 24 hr. per day and the other 1 hr. per day, it is obvious that the generating station should not be half way between but, rather, nearer the load with the high load factor. From a practical standpoint, the location theoretically best is rarely attainable. There are many other important considerations such as transportation facilities for fuel, available supply of cooling water, practicable building sites, etc., which govern the choice. It might be said that, with the present-day design of stations and the large capacities being attained, the matter of available water supply is becoming all important. The question of generating energy at the mouth of coal mines and transmitting to distant points has been much talked about recently. Such a practice seems at present, to be limited, however, to exceptional cases where an ample water supply may be had. In such cases

economy is realized if the energy can be so generated and transmitted to the point of utilization at a less cost per year, including investment charges and cost of losses on transmission lines, than the cost of the same energy generated at the feeding point, including the cost of transporting the coal by freight. In any case the economy of any location for a generating station is determined only by a complete study of the annual costs of all alternatives.

The location of a substation is a somewhat similar problem. In this case, however, the limiting factors are usually not so many, and the choice may be made on a more theoretical basis. A careful consideration of the loads to be carried, the lengths of the necessary lines, both underground and overhead, and the energy losses involved, will usually be very profitable. It often occurs that the expenditure of a little more money in the purchase of the best possible site may be repaid several times over in the economies effected on the distribution system.

The previous chapters in this book have discussed the problems pertaining to each part of the system with very little regard for the relation of that part to the system as a whole. If the study of the economics of the distribution system is to be made complete, this inter-relation must be considered. For example, we may determine the most economical drop in voltage on a transmission line and also on a power line. However, in order to properly serve the customer, the voltage regulation must be maintained to a certain standard. The problem is, then, to determine the most economical design and arrangement of transmission lines, substation, power lines and regulators to accomplish the desired result. This may possibly be somewhat different than the solutions for the individual parts. Such a study involves a careful determination of annual charges on all types of construction and a complete computation of the cost of energy loss.

Immediate results should not necessarily be expected from an application of engineering economics to an established distribution system. In some cases the money saved on one or two lines may prove the work well worth while. In other cases, the results will be evident only after some length of time. It is rare that a system can be brought up to an economical standard in a short time. Usually the engineer must be content to improve conditions gradually, by designing new extensions and rebuilding old work, here and there, in accordance with economi-

cal principles. The final results will be shown in the improved efficiency on the system as a whole. Often, this will take the form of a reduction in overall losses on the system, although this is not necessarily the case. The economical percentage of loss will depend on the relation between construction costs and the cost of energy, which may be very different for different systems. Hence the percentage of energy lost is not a true measure of economy. The ideal condition of maximum efficiency to which it is the purpose of all economic study to contribute, is that conditions in which every customer is provided with a reasonably good quality of service at the least possible cost over the whole system.

CHAPTER XVII

INDUSTRIAL PLANT PROBLEMS

Application to Industrial Plant Problems of the Principles of Economics Outlined for Electrical Distribution

This book has dealt primarily with electrical distribution problems from the point of view of the central station. The purpose, in general, has been to indicate means of studying a distribution system with the view to transmitting electrical energy from the generating plant to the customer at the least possible over-all cost. The second part of the book so far has dealt entirely with the problems encountered in the various parts of such a distribution system. The consumer, however, is interested but indirectly in such problems, in that a reduction in central station costs may lead to lower rates or better service. The consumer of any considerable amount of energy, however, such as a large industrial plant, has numerous problems of his own which may be classed as problems of electrical distribution. These may be viewed from the standpoint of the producer or the buyer of electrical energy according to whether he produces his own energy or buys it from the central station. In some cases these problems are very similar to those of the central station. In other cases they may be quite different. In any case, however, where the question of economy enters, the principles explained in Part I will be found fundamental. Annual cost is the basis of comparison for any alternative propositions. In general, the solution of many of the problems may be simplified by the use of a general equation such as that described in Chapter VI. The most economical condition will be discovered, only if all items of expense including the cost of energy losses as well as the fixed charges on investment are considered.

Voltage Regulation.—The problem confronting the electrical engineer in an industrial plant is, essentially, the same as that of the distribution engineer as stated in Chaper I, *i.e.*, to realize the greatest economy possible consistent with good service. In this case good service depends upon two factors. In the first

place, the central station must furnish reasonably good regulation at the consumer's service. This is usually more or less regulated by contract but may depend somewhat on the character of load imposed by the consumer as to power factor, fluctuation, etc. A customers load may be of such a nature that it is practically impossible to give good regulation at his service and other services on the same line may be similarly disturbed. In such a case it is usually necessary for the customer to change his equipment or method of operation in some way so as to remedy the difficulty. The second factor is the interior distribution of the plant itself. Assuming that good regulation is furnished by the central station, it is essential that the arrangement of the plant electrical distribution be such that good voltage conditions are maintained at the points of utilization. By plant distribution is meant all parts of the electrical circuit including that of apparatus of utilization. All such problems may at first be more electrical than economic. However, good service being accomplished, it is then essential to investigate the matter of economy.

Problems of Power Distribution.—The problems pertaining to the actual wiring in an industrial plant will be very similar to those of the larger distribution system which has been heretofore described. For small plants it will be simply a case of secondary distribution. The sizes of conductor can be readily determined by a consideration of the annual charges on the cost of the conductor in place versus the cost of losses. In the case of the consumer, the unit cost of lost energy is more easily determined than for the central station as it appears very decidedly in his monthly bills. For larger plants, with several separate buildings, it may be advisable to distribute partly at primary voltage. For still larger ones, generating their own power, the problem of high voltage transmission may enter. In any such case the problems of the central station are more nearly approached and the examples cited in the preceding chapters will apply.

In many plants the question arises as to the comparative advantages of electrical distribution of power as compared with mechanical, *i.e.*, individual motors on every machine instead of large motors with mechanical transmission to a group of machines. Of course, there are many factors in such a problem, including the type of machines used and method of operation. In general a study of the total annual cost of operation including fixed charges, electrical and mechanical losses, and a consideration

of any possible difference in production, labor costs, maintenance, etc. will be of great value in determining the proper choice. Such a study will be based on the principles explained in Part I.

Problems of Equipment.—The choice of proper equipment may have considerable effect on the economy of plant operation. For customers buying energy at primary voltage there is first the selection of proper primary switch house equipment. Transformers must be selected with the view of probable increase in load. However, if too large units are installed, not only is there a waste of investment but the additional core losses and the effect on the power factor may be important. It will sometimes be found economical in such a case to use an open delta installation until the load justifies the installation of the third unit. Where reserve transformers are required in case of trouble where a shutdown would be serious, the choice of the size of units will be influenced by that fact. In any such case as well as in the selection of other equipment such as switches, etc., a study of the total annual cost is essential.

A very common fault in the selection of apparatus of utilization is that of overmotoring. Where careful engineering has not been done, the tendency often is to install larger motors than are necessary for the use required with the idea that the reserve power obtained is advantageous. The investment charges in such a case are larger than necessary. Also, any considerable underloading of induction motors brings down the power factor and this may have considerable effect on the regulation. In case poor power factor is penalized in the rates it is important to maintain as good a value as possible. A careful study of the diversity factor between machines on the same motor is important in its proper size. In some cases synchronous motors operated to improve power factor or static condensers will be found advisable. Similar considerations will be encountered with other types of equipment. Old inefficient equipment may often be replaced to advantage, although it must be kept in mind that beyond a certain point efficiency is not always economy. In all cases, the annual cost must always be considered as an important factor along with probably load increase, regulation, reserve power, etc.

Problems of Operation.—In this country the establishment of rates modified by power factor considerations is becoming more and more important and is at present receiving considerable attention from the central stations. Poor power factor not only

occasions additional power losses all the way back to the generators but it increases the difficulties of maintaining good regulation. It would therefore seem just that a customer with a low power factor should pay more per kilowatt hour than one with a high power factor. With this in mind it is essential that every user obtain as good a power factor as possible from his plant. This may be done by a selection of equipment of proper size and type for the load as mentioned above and also by the use of condensers, either static or synchronous. The expenditure justified in order to increase power factor can be determined by a study of the reduction in the power bills so effected. In this way it will be discovered whether it will be a paying proposition to establish a power factor of say 95 per cent or of 80 per cent, etc.

The plant load factor will usually be an important consideration in a study to increase economy. At present most power rates are based on some form of demand charge and some form of kilowatt hour charge. It is obviously advantageous to reduce the demand charge if possible, spreading the energy used at the peak through the rest of the day. This is especially true where the rates are of such a form as, the demand load for "a" hours at "b" cents per unit and the reminder at "c" cents. The demand may often be reduced by a careful study of the time of starting up motors and of overlapping operation of various equipment. In some cases the reduction in the annual bills will warrant the purchase of new equipment to reduce the demand.

Other Problems.—It is not within the province of this book to attempt to discuss in any detail all problems in electrical distribution which occur in an industrial plant. It has been attempted above to outline some of the questions encountered and give a general idea of the field of application of economic principles to such problems. There are many others which are less strictly electrical or not at all electrical which will bear a similar study. The questions of proper illumination as affecting production, safety devices, labor saving devices, etc. are all more or less economic problems. The main point must be always kept in mind that, once good service is accomplished, the greatest advantage lies with any installation for which the total annual cost, everything considered, is less than any other.

APPENDIX A

METHOD OF APPROXIMATING ENERGY COST

It very often happens that it is desirable to study the economics of different types of circuits before it has been possible to undertake any detailed determination of energy cost as outlined in Chap. IV. It has been customary, with most engineers doing such work, to assume in such a case, an approximate average figure such as .01 per kilowatt-hour or some other figure which is known to be somewhere near the average for the whole output of the system. A little closer approximation may be made, keeping the principles set forth in Chap. IV in mind, by determining roughly the variation in cost with the load factor for the class of load being considered. While not very accurate, such figures are at least better than a rough guess of a cost to cover all cases.

An example of such a determination will be worked out here to show how it is possible to handle such a problem. The general methods used may be adopted to other similar cases.

The loads considered are power loads in the suburban districts, *i.e.*, such that considerable transmission-line cost is involved. As was explained in Chap. IV, the demand cost for energy losses is very nearly proportional to the amount of load at time of station peak. It was assumed that the average demand cost at the generating-station switchboard equals \$15.76 per kilowatt and the average kilowatt-hour cost = \$ .0036 per kilowatt-hour. (These costs are usually available and fairly correct in most companies.) The demand cost per kilowatt-hour will be inversely proportional to the load factor, considering load at generating station only. Reducing the demand cost to a cost per kilowatt-hour:

At 100 per cent load factor, demand charge $= \dfrac{15.76}{8760} =$ .0018 per kilowatt-hour.

For any other load factor, demand charge $= \dfrac{.0018}{\text{Load factor (expressed as a fraction not as per cent.)}}$

In order to determine the energy cost at any point on the

system beyond the generating-station, the costs given above must be increased by the cost of transmission and distribution. Also diversity factors must be taken into account.

A study of the property classification of the system indicated a division approximately as follows:

	Per cent
Generating plant	40.0
Underground conduit, etc	6.5
Poles and fixtures	7.0
Transmission lines	24.5
Distribution	19.3
Transformers	2.7

We may then assume, roughly, that the demand charge exclusive of the generating plant is 60 per cent and of the generating plant 40 per cent of the total. For a load of 100 per cent load factor the total demand charge would then be simply =

$$\frac{\text{generating-station demand charge}}{.40}$$

= 2.5 generating-station demand for 100 per cent load factor.

For any other load factor, the diversity between loads will affect the cost. A kilowatt load at the customer will not mean a kilowatt at the generator unless the peaks happen to coincide. The diversity factors were assumed in this case to be those given in the "Standard Handbook" for general power loads as follows:

	Per cent
Between transformers	74
Between power lines	87
Between substations	91

It is assumed that these factors hold good for a load factor of 25 per cent. Then, 1 kw. at the customer becomes

$1 \times 1 = 1$ kw. at the transformer.
$.74 \times 1 = .74$ kw. on the power line.
$.87 \times .74 = .61$ kw. at the substation.
$.91 \times .61 = .586$ kw. at the generating-station.

These ratios combined with the percentages of the total property values as given above, give the approximate share of each kilowatt at the customer in the total demand charge for the system. For example, if the transformer investment is 2.7 per cent of the total, and 1 kw. at the customer represents 1 kw. at

the transformer, that kilowatt should take 2.7 per cent of the total demand charge for its share in the transformer investment. If it becomes only .586 kw. at the generating station and the generating station represents 40 per cent of the total investment, the kilowatt in question should be charged with 23.5 per cent of the total demand charge for its share in the station. In order to make property and diversity factor classifications coincide and as an approximation, it was assumed that the investment in distribution and poles and fixtures could be taken to represent power lines, and that the investment in transmission and underground could be applied to substations.

Then for 1 kw. at the customer

	Per cent total investment	Share in total demand cost
On transformer..........	1.0 kw. × 2.7	2.7
On power line...........	.74 kw. × (19.3 + 7.0)	19.5
At substation............	.64 kw. × (6.5 + 24.5)	19.8
At generating station.....	.586 kw. × 40	23.5
Total.................		65.5

Then if total demand for any load factor

$$= \frac{\text{gen.-sta. demand for that load factor}}{.40}$$

The demand charge for 1 kw. at the customer with the load factor assumed

$$(25 \text{ per cent}) = \frac{\text{gen.-station demand at 25 per cent load factor} \times 0.655}{.40}$$

$$= 1.63 \times \text{gen.-station demand at 25 per cent load factor}$$

From the two values thus obtained for 100 per cent and 25 per cent load factors and the point for load factor equals zero, the curve shown on Fig. 58 was plotted showing by what amount the generating station demand cost at any load factor should be multiplied to give the demand charge at the customers in question.

Similarly the kilowatt-hour charge must be increased to account for losses in transmission and distribution. It was assumed that the total loss between generating station and customer would be not far from 30 per cent and, for the loads considered, it was assumed that it would be accurate enough to increase the generating-station, kilowatt-hour charge by 30 per

cent to give the charge at the customer. The average kilowatt-hour charge would then = .036 × 1.36 = .0047.

The following table may then be derived. The first column gives the average number of hours of operation per week (t_w) at

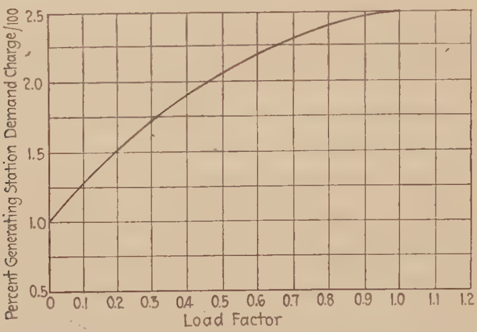

Fig. 58.—Three-phase secondary. Factor by which demand charge at generating station for any load factor must be multiplied to obtain demand charge at load.

full load corresponding to any load factor. The last column gives the corresponding values of t_wC_e. These are used as shown in Chaps. XI and XIV. The values of t_wC_e are plotted in Fig. 59

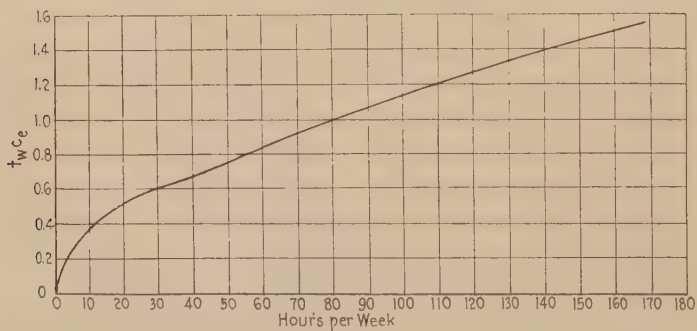

Fig. 59.—Values of t_wC_e for different values of t_w (hours per week).

The cost of energy obtained in this way is very approximate but may serve the purpose until some more accurate determination can be made. The figures here given must be considered as examples only and not as representative of present-day costs on any system.

TABLE 20

t_w	Load factor	Station demand	Multi-plier	Demand at load	Total at load	t_wC_e
.0	.00	.0	1.00	.00	.0	.0
16.8	.10	.018	1.28	.0231	.0278	.467
33.6	.20	.009	1.52	.0137	.0184	.619
50.4	.30	.006	1.72	.0103	.0150	.756
67.2	.40	.0045	1.90	.0086	.0133	.894
84.0	.50	.0036	2.05	.0074	.0121	1.015
100.8	.60	.003	2.20	.0066	.0113	1.139
117.6	.70	.0026	2.32	.0060	.0107	1.258
134.4	.80	.00225	2.41	.0054	.0101	1.358
151.2	.90	.002	2.47	.0049	.0096	1.450
168.0	1.00	.018	2.50	.0045	.0092	1.543

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